所別：工業管理研究所碩士班 乙組 科目：作業研究

1. True or False (2 points each)
   a. Every global optimum in LP is a local optimum.
   b. Infeasible solution is a solution for which at least two constraints are violated.
   c. The only possibility for a LP problem having no optimal solution is that the problem has no feasible region.
   d. Every optimal solution to a LP problem will be a boundary point of its feasible region.
   e. The shadow price is the value of one additional unit of a scarce resource.

2. Consider a typical LP problem in matrix form.

$$\text{Maximize } z = cx$$
$$\text{Subject to } Ax \le b$$
$$\text{and } x \ge 0,$$

where A is a $m \times n$ matrix, x is a column vector of size n, c is a row vector of size n, b is a column vector of size m. Show that the feasible region of the above problem is a convex set. (10 points)

3. Consider a taxi station where taxis looking for passengers, taxis arrive according to Poisson process with rate 1 per minute and passengers arrive according to Poisson process with rate 1.25 per minute. A taxi will wait no matter how many other taxis are in line; but an arriving passenger waits only if the number of passengers already waiting for taxis is three or less. Assuming steady-state conditions,

a. Define the state for this problem. (5 points)
b. Draw the transition diagram and compute the steady-state probability for each state. (8 pints)
c. Find the expected number of taxis waiting for passengers (7 points)
d. Find the expected number of passengers waiting for taxis. (5 points)

4. Consider the following nonlinear programming problem.

$$\text{Maximize } z = 2x_1^2 + 2x_2 + 4x_3 - x_3^2$$
$$\text{Subject to } 2x_1 + x_2 + x_3 \le 4$$
$$\text{and } x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.$$

We want to use dynamic programming to solve this problem.

a. Define the state variables and decision variables for each stage. (5 points)
b. Solve the problem by dynamic programming. (15 points)

參考用

注意：背面有試題

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5. A fugitive moves between two cities, Taipei (TP), and Hong Kong (HK), to escape a bounty hunter. Initially, the fugitive is in Taipei and the bounty hunter is in Hong Kong. The fugitive and the bounty hunter move independently of each other and each of them follow the respective transition probability matrix of a Markov chain as follows:

$$P_{(\text{fugative})} = \begin{array}{c} \\ TP \\ HK \end{array}\begin{array}{c} \begin{array}{cc} TP & HK \end{array} \\ \begin{bmatrix} 0.4 & 0.6 \\ 0.7 & 0.3 \end{bmatrix} \end{array}$$

$$P_{(\text{bounty hunter})} = \begin{array}{c} \\ TP \\ HK \end{array}\begin{array}{c} \begin{array}{cc} TP & HK \end{array} \\ \begin{bmatrix} 0.5 & 0.5 \\ 0.8 & 0.2 \end{bmatrix} \end{array}$$

For example if the fugitive is in Taipei at the end of one day, the probability he is in Taipei at the end of the next day is 0.4. We assume that each person makes the move (transition) at the end of each day. When the two are in the same city on a given day there is a 50 percent chance the fugitive will be captured. Our primary goal is to compute the expected time for the bounty hunter to catch the fugitive.

a. Define the state for this problem. (5 points)

b. Compute the new transition matrix P. (8 points)

c. Compute the expected time for bounty hunter to catch the fugitive. (12 points)