國立中央大學九十二學年度碩士班考試入學招生試題卷

系所別: 企業管理學系 甲組 戊組 科目: 統計學

參考用

1. Consider the regression line $Y_i = \beta X_i + \varepsilon_i$, i=1, 2, … , n, that is,

   $E(Y) = 0$ when x = 0. Assume that $E(\varepsilon_i) = 0$, $\text{Var}(\varepsilon_i) = \sigma^2$.

   Derive the least square estimate for β. (10%)

2. Develop a linear piecewise model with changing point at X=6 to estimate the following data: (10%)

Y	10.2	9.3	9.4	8.5	8.6	8.0	6.5	5.8	4.0	3.5	1.9
X	1	2	3	4	5	6	7	8	9	10	11

3. If you want to investigate the difference in MBA's initial salary between public and private business school, which affected by the factors of established years, number of alumni, and enrollment.
   (a) State your hypothesis and propose suitable statistic to test your hypothesis. (10%)
   (b) How to further verify if the former undergraduate background of MBA in sciences, engineering, business, or humanities could affect their initial salary. (10%)

4. We have sales data of five brands from nine supermarkets, $\overline{X}_1, \ldots, \overline{X}_5$ and $s_1^2, \ldots, s_5^2$,

   construct the $100(1-\alpha)\%$ confidence interval for $\frac{1}{2}(\mu_1 + \mu_2) - \frac{1}{3}(\mu_3 + \mu_4 + \mu_5)$.

   (15%)

5. Only one in 1000 adults is afflicted with a rare disease for which a diagnostic test has been developed. The test is such that, when an individual actually has the disease, a positive result will occur 99% of the time, while an individual without the disease will show a positive test result only 2% of the time. If randomly selected individual is tested and the result is positive, what is the probability that the individual has the disease? (10%)

6. The weekly demand for propane gas (in thousands of gallons) from a particular facility is a random variable with p.d.f.

   $$f(x) = \begin{cases} 2\left(1 - \dfrac{1}{x^2}\right) & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the median estimate. (10%)
   (b) If 1.5 thousand gallons is in stock at the beginning of the week and no new supply is due in during the week, how much of the 1.5 thousand gallons is expected to be left at the end of the week? (10%)

7. Suppose that my waiting time for a bus is uniformly distributed on $[0, \theta]$, and that the results $X_1, \ldots, X_n$ of a random sample from this distribution have been observed.
   (a) Sketch the likelihood function for $f(X_1, \ldots, X_n; \theta)$ (8%)
   (b) Derive the Maximum Likelihood Estimator for θ. (7%)