所別：企業管理學系碩士班 一般類組(乙組) 科目：工程數學

1. True or False (20 pts.)
   (4 pts. for each. A wrong answer will result in 2 more pts. loss!)

   (1) The general solution of an N^{th}-order differential equation, no matter linear or nonlinear, will have N arbitrary constants.
   (2) Any linear combination of two solutions of a 2^{nd}-order non-homogeneous linear differential equation is also its solution.
   (3) The equation $\{\sin(x)\cosh(y)\}-\{\cos(x)\sinh(y)\}y'=0$ is exact.
   (4) For the problem: $y''+P(x)y'+Q(x)y=0$ with $y(x_0)=K_0$ and $y(x_1)=K_1$, $P(x)$ and $Q(x)$ are continuous on an open interval I, its solution is always unique.
   (5) The general solution of $y''+P(x)y'+Q(x)y=0$, where $P(x)$ and $Q(x)$ are continuous on some open interval, is always of the form: $y(x)=c_1y_1(x)+c_2y_2(x)$. $y_1(x)$ and $y_2(x)$ form a basis. Singular solutions do not exist.

2. Please solve $\dfrac{dy}{dx}=\dfrac{2x\tan y}{\sec^2 y}$ (10 pts.)

3. Given the solutions $y_1(x)=\cos(\ln x)$ and $y_2(x)=\sin(\ln x)$

   (1) Find Wronskian: $w(y_1,y_2)$ (3 pts.)
   (2) Find the corresponding 2^{nd}-order homogeneous linear differential equation (7 pts.)

4. Find the inverse Laplace transform of $\dfrac{\pi^5}{s^4(s^2+\pi^2)}$ (8pts.)

5. Please first solve $xy'-2y=2$ and then use the method of power series to verify your answer. (10 pts.)

6. Please solve $y''+2y+2=4e^{-x}\sec^3 x$. (10 pts.)

7. Please solve y(x) in the following equation:

$$y(x)=x^5+x\int_0^x\{\omega y(\omega)\}\,d\omega$$ (10 pts.)

注意：背面有試題

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8. Please convert the following 3rd-order differential equation into a differential system and then solve it

$y''' + 2y'' - y' - 2y = 0$ (10 pts.)

9. Please answer the following two questions:

(1) Use the reduction of order to find the other solution, y_2, of the differential equation: $xy'' + 2y' + xy = 0$, given $y_1 = \dfrac{\sin x}{x}$ (6 pts.)

(2) Let $y_1(x) \neq 0$ and $y_2(x)$ be two linearly independent solutions of $P_0(x)y'' + P_1(x)y' + P_2(x)y = 0$

Show that $y(x) = \dfrac{y_2(x)}{y_1(x)}$ is a non-constant solution of the following differential equation:

$$y_1(x)y'' + \left(2y_1'(x) + \frac{P_1(x)}{P_0(x)}y_1(x)\right)y' = 0$$

(6 pts.)