Instructions: Answer the following questions. Make and state your own assumptions for questions where the information is not sufficient for you to solve them. For example, if you need the corresponding p-value of a normally distributed random variable evaluated at 2.5, you may indicate the value as, say, $Pr(x \geq 2.5)$, where $x \sim \mathcal{N}(0,1)$.

1. (30 points) If the stock price p_t follows an *iid* random walk process as follows:

$$p_t = p_{t-1} + e_t, t = 1, ..., T;$$

where e_t has an *iid* normal distribution, i.e., $e_t \stackrel{iid}{\sim} \mathcal{N}(0,\sigma^2)$. Let $r_t = p_t - p_{t-1}$ be the stock return. Define the following random variable:

$$I_t = \begin{cases} 1 & \text{if } r_t \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Also define the following two random variables:

$$Y_t = I_t I_{t+1} + (1 - I_t)(1 - I_{t+1})$$
$$N = \sum_{t=1}^{T} Y_t$$

Answer the following questions:

(a) Calculate the variance of Y_t, $Var(Y_t)$.

(b) Calculate the first-order correlation coefficient of Y_t, $corr(Y_t, Y_{t+1})$.

(c) Calculate the variance of N, $Var(N)$.

2. (20 points) Let X and Y have the following joint density.

$$Pr(X = x, Y = y) = c, 0 \leq y \leq x \leq 1. \tag{2}$$

(a) (10 points) Find c.

(b) (10 points) $E(Y|X \geq \frac{1}{2})$.

3.

The data, presented below, represent a sample of advertising expenditures x and the associated sales volumes y for a company during 10 randomly selected months.　We wish to determine whether advertising is actually worthwhile.

y_i (in \$10,000)	x_i (in \$10,000)	x_i^2	$x_i y_i$	y_i^2
101	1.2	1.44	121.2	10201
92	.8	.64	73.6	8464
110	1.0	1.00	110.0	12100
120	1.3	1.69	156.0	14400
90	.7	.49	63.0	8100
82	.8	.64	65.6	6724
93	1.0	1.00	93.0	8649
75	.6	.36	45.0	5625
91	.9	.81	81.9	8281
105	1.1	1.21	115.5	11025
Sum 959	9.4	9.28	924.8	93569

(a) Obtain the least squares prediction line for the data.　What would be the predicted sales volume if the company has budgeted \$10,000 for advertising in a month? (10%)

(b) Calculate an estimate of σ^2 for the data.　(5%)

(c) Is there evidence to indicate that the slope coefficient (β_1) differs from 0?　(the relevant critical value of t are: $t_{.025} = 2.306$, $t_{.005} = 3.355$)　(10%)

(d) Find a 95% confidence interval for β_1.　Find a 95% confidence interval for the expected monthly sales volume for an advertising expenditure of \$10,000.　(10%)

(e) Find a 95% confidence interval for the next month's sales if the advertising expenditure is \$10,000, assuming that other economic conditions remain approximately the same.　(5%)

(f) Use the analysis of variance procedure to test for a significant linear relationship between sales and advertising expenditures.　(the relevant critical value is $F_{.005} = 14.69$)　(10%)