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所別：財務金融學系碩士班 乙組 科目：微積分

1. (20%) Integral can be valued using numerical methods of integration. In this question, please use different numerical methods to approximate the value of a definite integral.

Evaluate the integral $\int_1^2 \frac{1}{x^2}dx$, by dividing the interval $[1,2]$ into 4 subintervals using

(a). (5%) Riemann sum;

(b). (5%) the Trapezoidal rule;

(c). (5%) the Simpson's rule;

(d). (5%) the exact value of the definite integral.

Round your answers to four decimal places.

2. (20%) Please evaluate the following indefinite integrals:

(a). (10%) $\int \frac{1-\sqrt{x}}{1+\sqrt{x}}dx$

(b). (10%) $\int \frac{1}{x^2\sqrt{4+x^2}}dx$

3. (15%) Please find the sum of the series:

(a). (7%) $\sum_{n=0}^{\infty}\left[\left(\frac{2}{3}\right)^n - \frac{1}{(n+1)(n+2)}\right]$

(b). (8%) $\sum_{n=0}^{\infty}\frac{2^n}{3^n n!}$

4. (20%) Please evaluate the following integrals:

(a). (10%) $\int_0^4\int_{\sqrt{x}}^2 dydx$

(b). (10%) $\int_0^2\int_y^{\sqrt{8-y^2}}\sqrt{x^2+y^2}dxdy$

5. (25%) The call option price at current time $t=0$ is $C_0 = S_0N(d_1) - Ke^{-rT}N(d_2)$, where $d_1 = \frac{\ln\left(\frac{S_0}{K}\right)+rT+\frac{1}{2}\sigma^2T}{\sigma\sqrt{T}}$, and $d_2 = d_1 - \sigma\sqrt{T}$. r, σ, T and K are constant. The function $N(x)$ is the probability that a standard normal random variable is less than x. For example, $N(a)$ is given by $N(a) = \int_{-\infty}^{a}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy$. This question asks you to derive C_0, the price of a call option at current time $t=0$ using the integration method. You don't necessarily need to know the definition of a call option when deriving the formulae for C_0. You just need to evaluate the following integral

$$C_0 = e^{-rT}\int_{-\infty}^{\infty}\max\left(S_0e^{Y_T}-K,0\right)f\left(Y_T\right)dY_T$$

where Y_T is a normally distributed random variable with mean $=\left(r-\frac{1}{2}\sigma^2\right)T$, and variance $=\sigma^2T$, and $f\left(Y_T\right)$ is the probability density function of random variable Y_T. S_0 is a known constant at current time $t=0$.

(Hint)(i) The probability density function of a normally distributed random variable z_T with mean$=\mu$, and variance$=\sigma^2$, is given by $f\left(z_T\right) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\left(\frac{z_T-\mu}{\sigma}\right)^2}$.

(ii) The function $\max(a,b)$ means taking the maximum value of the two arguments, that is $\max(a,b) = a$ if $a > b$.