Instructions: Answer the following questions. Make and state your own assumptions for questions where the information is not sufficient for you to solve them. For example, if you need the corresponding p-value of a normally distributed random variable evaluated at 2.5, you may indicate the value as, say, $Pr(x \geq 2.5)$, where $x \sim \mathcal{N}(0,1)$.

1. (25 points) Answer the following two questions:

   (a) (10 points) Suppose X and Y are two random variables such that the correlation coefficient $\rho(X,Y) = 1/2$, $Var(X) = 1$, and $Var(Y) = 2$. Compute $Var(X - 2Y)$.

   (b) (15 points) Let X be uniformly distributed on $\{0, 1, ..., N\}$. Find the mean and variance of X.

2. (25 points) Suppose a box has 3 balls labeled 1, 2, and 3. Two balls are selected. Let X be the number on the first ball and let Y be the number on the second ball. Compute $Cov(X,Y)$ and $\rho(X,Y)$ for the following two cases.

   (a) (10 points) The two balls are drawn with replacement.

   (b) (15 points) The two balls are drawn without replacement.

3. (25 points) Given $f(x|\theta) = 1/\theta$, $0 \leq x \leq \theta$, and given the hypotheses $H_0 : \theta = 1$ and $H_1 : \theta = 4$, suppose a single observed of X is to be taken.

   (a) (15 points) If the critical region is chosen to be the interval $X > 0.5$, what are type I error α and type II error β?

   (b) (10 points) What would those values become if $X > 1.5$ were chosen as the critical region?

4. (25 points) Consider the data set listed below:

Case	Language Aptitude Y	Analogical Reasoning X_1	Geometric Reasoning X_2
A	2	3	15
B	6	8	9
C	5	2	7
D	9	4	3
E	11	10	2
F	12	15	1
G	1	4	12
H	7	3	4

Suppose you wish to see if Y is linearly associated with X_1 and X_2.

   (a) (5 points) Compute the 3×3 product moment correlation matrix of Y, X_1 and X_2.

   (b) (5 points) Find the regression equation of Y on X_1 and X_2.

   (c) (5 points) Find R^2.

   (d) (10 points) Test for the significance of the coefficients of X_1 and X_2.