所別：財務金融學系碩士班 丙組 科目：微積分

1. (10%) Please find the area of region bounded by the graphs of $x = 4 - y^2$ and $x = y - 2$ by integrating with respect to x.

2. (10%) Please evaluate the integral $\int x^3 e^x dx$.

3. (10%) Please evaluate the following limit.

   (a) (5%) $\lim\limits_{x \to \infty} \frac{x}{\sqrt{x^2+1}}$

   (b) (5%) $\lim\limits_{x \to \infty} (\ln x)^{\frac{2}{x}}$

4. (10%) We put P dollars in a saving account for T years. The number of compoundings for interest is n times per year. At the annual interest rate is r, the total amount (i.e. principal plus interest) that one can receive after t years is $P\left(1 + \frac{r}{n}\right)^{nT}$. If the number of compoundings per year becomes infinite. i.e. $n \longrightarrow \infty$, what is the total amount that one can receive after T years? (You have to give the derivation to justify your answer.)

5. (10%) Please find the sum of the series $\sum\limits_{n=1}^{\infty} [(0.7)^n + (0.9)^n]$.

6. (10%) Please evaluate the integral of $\int_0^2 \int_x^2 e^{-y^2} dy dx$.

7. (25%) A random variable Q has the following form

$$Q = bX + cX^2$$

   where b and c are real constants, and X is standard normally distributed random variable with mean = 0, variance = 1, that is $X \sim N(0,1)$. The moment generating function of Q is given by $E\left[e^{tQ}\right]$. Please find $E\left[e^{tQ}\right]$, i.e. $E\left[e^{tQ}\right] = E\left[e^{t\left(bX+cX^2\right)}\right] = \int_{-\infty}^{\infty} e^{t\left(bX+cX^2\right)} f(X)\, dX$, where $f(X)$ is the probability density function of X, $f(X) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}X^2}$.

8. (15%) The current price of a financial product P is a function of variable S, r, K, T, and σ as follows:

$$P = SN(d_1) - Ke^{-rT} N(d_2)$$

   where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T} = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

   and $N(a)$ is the probability that a variable with a standard normal distribution will have a value less than a. $N(a)$ is given by $N(a) = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy$.

   Please find $\frac{\partial P}{\partial \sigma}$, i.e. the derivative of P with respect to σ.