

國立中央大學八十四學年度碩士班研究生入學試題卷

所別: 財務管理研究所

組 科目: 統計學

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(20%)1. A market research group specializes in providing assessments of the prospects of sites for new clothing stores in shopping centers. The group assesses prospects as either good, fair, or poor. The records of requests for assessments made to this group were examined, and it was found that for all stores that turned out to be successful, the assessment was good for 70%, fair for 20%, and poor for 10%. For all stores that turned out to be unsuccessful, the assessment was good for 20%, fair for 30%, and poor for 50%. It is also known that 60% of new clothing stores are successful and 40% are unsuccessful.

- (5%)a. For a randomly chosen store, what is the probability that prospects will be assessed as good?
- (5%)b. If prospects for a store are assessed as good, what is the probability that it will be successful?
- (5%)c. Are the events "Prospects assessed as good" and "Store is successful" statistically independent?
- (5%)d. Suppose that five stores are chosen at random. What is the probability that at least one of them will be successful?

(15%)2. A random sample, X_1, X_2, \dots, X_n , of n observations is taken from a population with mean μ and variance of σ^2 . Consider the following estimator of μ :

$$\hat{\mu} = (2/n(n+1))(x_1 + 2x_2 + 3x_3 + \dots + nx_n)$$

- (7.5%)a. Show that $\hat{\mu}$ is an unbiased estimator of μ .
- (7.5%)b. Find the efficiency of $\hat{\mu}$ relative to \bar{X} , the sample mean.

[Hint: $\sum_{i=1}^n i = (n(n+1))/2$ and $\sum_{i=1}^n i^2 = (n(n+1)(2n+1))/6$]

(15%)3. Suppose that time spent, in hours, by students studying for a test has a normal distribution. A random sample of six students found the following results for hours spent studying

12.2 18.4 23.1 11.7 8.2 24.0

- (5%)a. Find a 99% confidence interval for the population mean.
- (5%)b. Find a 99% confidence interval for the population variance.
- (5%)c. Without doing the calculations, state whether a 90% confidence interval for the population mean would be wider than or narrower than that found in (b).

(10%)4. A company produces electric devices operated by a thermostatic control. The standard deviation of the temperature at which these controls actually operate should not exceed 2.0° Fahrenheit. For a random sample of twenty of these controls, the sample standard deviation of operating temperatures was 2.36° Fahrenheit. Stating any assumptions you need to make, test at the 5% level the null hypothesis that the population standard deviation is 2.0 against the alternative that it is bigger.

(20%)5. A wine producer claims that the proportion of its customers who cannot distinguish its product from frozen grape juice is at most 0.10. The producer decides to test this null hypothesis against the alternative that the true proportion is more than 0.10. The decision rule adopted is to reject the null hypothesis if the sample proportion who cannot distinguish between these two flavors exceeds 0.14.

- (6%)a. If a random sample of 100 customers is chosen, what is the probability of a Type I error, using this decision rule?
- (8%)b. If a random sample of 400 customers is selected, what is the probability of a Type I error, using this decision rule? Explain why your answer differs from that in part (a).
- (5%)c. Suppose that the true proportion of customers who cannot distinguish between these flavors is 0.20. If a random sample of 100 customers is selected, what is the probability of a Type II error?

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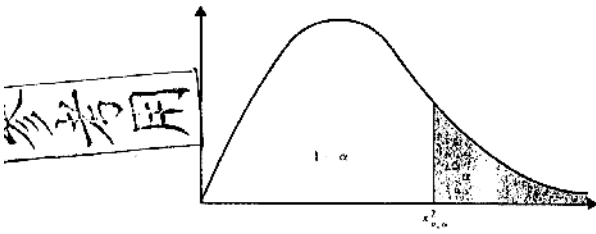
(20%)6. An executive of a prepared frozen foods company is interested in the amounts of money spent on such products by families in different income ranges. Independent random samples of six families with incomes under \$15,000 a year, five families with incomes between \$15,000 and \$30,000 a year, and four families with incomes over \$30,000 a year were taken. The estimates of monthly expenditures (in dollars) on prepared frozen foods given by the sample members are shown in the table.

under \$15,000	\$15,000-\$30,000	over \$30,000
45.2	48.2	50.7
60.1	51.6	71.6
52.8	63.7	61.3
31.7	46.8	49.8
33.6	49.2	
39.4		

- (8%)a. Set out the analysis of variance table.
- (6%)b. Test the null hypothesis that population mean expenditures on prepared frozen foods are the same for all three income groups.
- (6%)c. Use the Kruskal-Wallis procedure to test the null hypothesis of equal population mean expenditures on prepared frozen foods for the three income groups.

參考用

2. Cutoff points of the chi-square distribution function

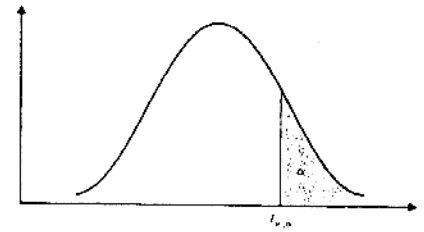


For selected probabilities α , the table shows the values x_{α}^2 such that $\alpha = P(x^2 > x_{\alpha}^2)$, where x^2 is a chi-square random variable with ν degrees of freedom. For example, the probability is .100 that a chi-square random variable with 10 degrees of freedom is greater than 15.99.

ν	α									
	.995	.990	.975	.950	.900	.100	.050	.025	.010	.005
1	0.00393	0.00157	0.00982	0.01393	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.51	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

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3. Cutoff points for the Student's t distribution



For selected probabilities, α , the table shows the values $t_{\alpha, \nu}$ such that $P(t > t_{\alpha, \nu}) = \alpha$, where t is a Student's t random variable with ν degrees of freedom. For example, the probability is .10 that a Student's t random variable with 10 degrees of freedom exceeds 1.372.

ν	α				
	.100	.050	.025	.010	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.364	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.972
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.331	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
∞	1.282	1.645	1.960	2.326	2.576

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