

# 國立中央大學八十八學年度碩士班研究生入學試題卷

所別: 產業經濟研究所 甲組 科目: 微積分 共 1 頁 第 1 頁

- (10%) Let  $f'(x) = 5f(x) \cdot (1 - \frac{f(x)}{10})$  and  $A(1, 6)$  be a point of the graph of  $f$ . Find the equation of the tangent line to the graph of  $f$  at the point  $A$ .
- (10%) Suppose  $\int_1^x f(t)dt = xe^{\sin x} - (\frac{x}{1+x^2})^2 + 1$ . Find  $f(t)$ .
- (10%)
  - Prove that, for every real number  $x$ ,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

- Approximate  $\int_0^1 e^{-x^2} dx$  to three decimal places.
- (15%) 開車往返於台北與中壢的高速公路上, 如果車況良好 (即車的速率分別是時間的平滑函數) 且往返所花的時間相同為  $T$  分鐘, 試說明
    - 存在一時間  $t_0 \in (0, T)$ , 使得往返各花  $t_0$  分鐘時, 其速率相同.
    - 在台北與中壢間, 存在同一地點, 使得往返此地點時, 其速率相同.

- (15%) Find the following integrals:

- $\int_1^2 (2^x + x^2)^2 dx$

- $\int_0^1 x^3 e^{2x} dx$

- $\int \frac{1}{x^2+6x+16} dx$

- (15%) Show that a tin can of specified volume  $K$  will be made of the least amount of metal if its height equals the diameter of its base. (即, 試證明: 一圓柱鋁罐, 當體積給定為  $K$  時, 其最省材料的製造是, 高度為其底部之直徑.)
- (10%) Find the relative extrema of the function  $f(x) = \frac{x^3+27}{x^2-9}$  and sketch its graph.
- (15%)
  - Find the local maxima, local minima, and saddle points of

$$z = (x^2 - y^2)e^{-\frac{x^2+y^2}{2}}$$

- The density of a metallic spherical surface  $x^2 + y^2 + z^2 = 4$  is given by  $\rho(x, y, z) = 2 + xz + y^2$ . Find the places where the density is highest and lowest.

參考用