

系所別： 產業經濟研究所 甲組 科目： 統計學

1. A random variable  $X$  follows the exponential distribution if  
 (15%) it has the following probability density function.

$$f(x) = (\frac{1}{\theta}) e^{-x/\theta} \quad \text{for } x > 0 \\ = 0 \quad \text{elsewhere}$$

where  $\theta > 0$  is the parameter of the distribution. Using the maximum likelihood method to find the estimator of  $\theta$ .

2. Given the following model  
 (25%)

$$Y_t = \beta_1 + \beta_2 X_{t+1} + \beta_3 X_{t+2} + \dots + \beta_k X_{t+k} + \varepsilon_t$$

where  $\varepsilon_t \sim N(0, \sigma^2)$ . Under the null hypothesis that  $\beta_2 = \beta_3 = \dots = \beta_k = 0$ , how do you perform the test?

3. Define  $h(t)$  for  $|t| \leq 1$  by  $h(t) = \sum_{n=0}^{\infty} t^n P(X=n)$   
 (60%)

(a). Show that  $h(t)$  is the factorial moment generating function for some r.v.  $X$ .

(b). Find the  $n$ -th factorial moment of  $X$ .

(c). Find  $E X^3$ .

(d). Find the factorial moment generating function for  $Y = 3X + Z$ .

