國立中央大學八十五學年度碩士班研究生入學試題卷

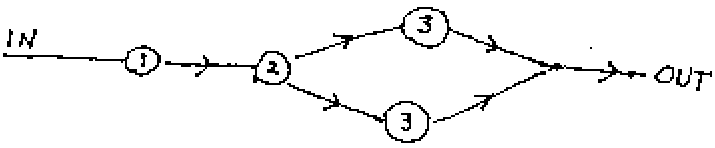
所別：產業經濟研究所 甲組 科目：甲統計學 共 2 頁 第 1 頁

1. (20%) Let a point be chosen uniformly from the interior of a triangle having a base of length l and height h from the base. Let X be defined as the distance from the point chosen to the base. Find the distribution function of X.

2. (40%) A system with three components 1, 2, and 3 is successful only if none of the components fail at a given moment. The failure probabilities of the three components are $p_1 = 0.1$, $p_2 = 0.2$, and $p_3 = 0.4$ respectively, and the components fail independently of each other.

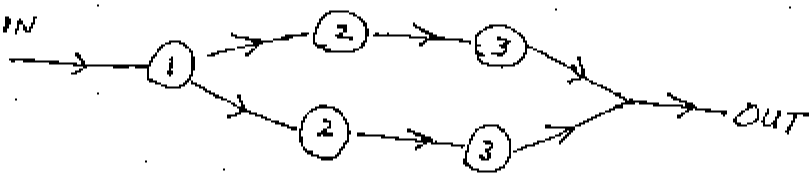
   (a). (10%) What is the probability of the system being successful?

   (b). (10%) What is the probability of the system being successful if the last component is duplicated as in Figure a below?

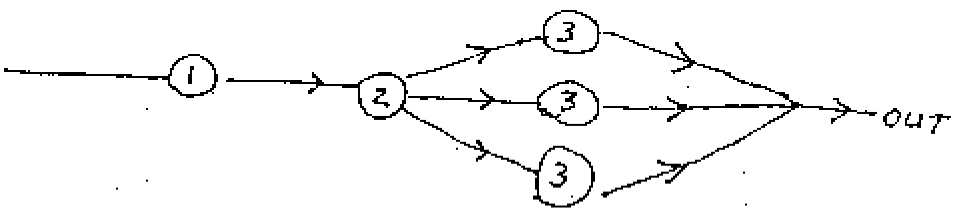


(Figure a)

   (c). (20%) If we are permitted a total of 5 components, is it better to duplicate both the second and third components (Figure b below) or to have the last component in triplicate (Figure c below)?



(Figure b)



(Figure c)

3. (20%) Suppose in a hypothetical market, only two brands 0 and 1 of an item are available. Assume that a customer buys an item (either 0 or 1) every week and his choice of brand in any week depends only on the brand he bought in the previous week and not on his choice of brand in any other week. Thus, if X_n denotes his choice of brand in the n-th week, then X_n can be either 0 or 1 and for $j=0,1$, $P(X_n=j \mid X_{n-1}=i_{n-1}, \dots, X_1=i_1) = P(X_n=j \mid X_{n-1}=i_{n-1})$ where each of $i_1, i_2, \dots, i_{n-1}$ is either 0 or 1, and $n \geq 2$. Assume also that $P(X_n=j \mid X_{n-1}=i)$ depends only on i and j and is independent of the week n, i.e. these probabilities do not vary over the weeks. For $n \geq 2$, write $P_{ij} = P(X_n=i \mid X_{n-1}=j)$, where $i=0,1$, and $j=0,1$. Obtain, in terms of P_{ij}'s, the probability that the customer who bought brand 1 in the first week will buy the same brand in the third week also.

4. (20%) A box has 3 red balls and 2 black balls. A random sample of size 2 is drawn without replacement. Let U be the number of red balls selected and let V be the number of black balls selected. Compute $\rho(U,V)$. Note that $\rho(X,Y) = \dfrac{Cov(X,Y)}{\sqrt{Var(X)\cdot Var(Y)}}$.

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