所別：經濟學系碩士班 科目：個體經濟學

The entrance examination consists of four questions. Each question contains a series of sub-questions. Each sub-question is given some points depending on its importance. Good luck!

A. (40 pts) The basic problem facing a typical consumer is to maximize her utility, subject to limited resources. Answer the following questions.

Given the utility function $U(x_1, x_2) = (x_1 + 3)(x_2 + 2)$ and the constraint $p_1x_1 + p_2x_2 = M$:

1. (3 pts) Write the Lagrangian function.
2. (5 pts) Find the optimal levels of x_1^*, x_2^*, and λ^* in terms of p_1, p_2, and M.
3. (7 pts) Find all the comparative-static derivatives(e.g. $\partial x_1^*/\partial p_1$), evaluate their signs, and interpret their economic meanings.
4. (5 pts) Check the second-order sufficient condition for maximum by the bordered Hessian $|\overline{H}|$.
5. (5 pts) Explain in detail how to apply the implicit-function theorem to the comparative-static analysis.

Now suppose the consumer allocates time as well as money income to different activities, receives income from time spent working in the labor market, and receives utility from consuming goods and participating in activities (e.g. eating, sleeping, watching television, travelling). The utility function can be extended to

$U = U(x_1, x_2, t_1, t_2)$,

where x_1 and x_2 are goods bought from the product market and t_j is the time spent at the jth activity. A time-budget constrait joins:

$t_1 + t_2 + t_w = T$,

where T is the total time available and t_w is the time spent working for pay. And the money-income constraint now becomes:

$p_1x_1 + p_2x_2 = wt_w$,

where w is the prevailing wage rate in the labor market.

6. (5 pts) Combine these two constraint into one in a full-income perspective and interpret its economic meaning.
7. (5 pts) Write the Lagrangian function and derive all the first-order conditions.
8. (5 pts) Write all the equilibrium conditions in terms of the marginal rate of substitution between goods, between time and good, and between different uses of time.

B. (10 pts) Suppose a consumer has a Bernoulli utility function $u(x) = \sqrt{x}$ and has an initial wealth 45.

1. (5 pts) If she owns a lottery which offers a payoff of 36 with probability $\frac{2}{3}$ and a payoff 4 with probability $\frac{1}{3}$. What is the minimum price she would sell it for?
2. (5 pts) If she does not own the lottery described in Question (i), what is the maximum price she would be willing to pay for it? Note that you do not have to solve the precise value in this question, just write a concise equation.

注意：背面有試題

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C. (30 pts) Suppose that there are only two firms, say Firms 1 and 2, on the market of a homogeneous and infinitely indivisible commodity. Firm i produces an amount y_i of the commodity for $i = 1, 2$. Let $Y = y_1 + y_2$ be the total output of the commodity. Therefore, the market price is taken to be $p(Y) = p(y_1 + y_2)$. Furthermore, suppose that each one of them is equipped with a cost function $c_i(y_i)$ for $i = 1, 2$.

1. (5 pts) Provide the formal definitions of the Cournot model and a Cournot (or Cournot-Nash) equilibrium. Describe the Cournot model as an extensive form game.
2. (2 pts) Assuming an interior optimization for each firm. Provide necessary conditions for an Cournot equilibrium in this framework.
3. (5 pts) Provide the formal definitions of the Stackelberg model and a Stackelberg equilibrium. Describe the Stackelberg model as an extensive form game.
4. (3 pts) Again assuming an interior optimization for each firm. Provide necessary conditions for an Stackelberg equilibrium in this framework.
5. (15 pts) What do we mean by an iso-profit curve for a firm in this framework? Describe the relationship between a Cournot equilibrium and a Stackelberg equilibrium in terms of reaction curves and iso-profit curves. Can you find a way to generalize the Cournot and Stackelberg models? Describe your idea.

D. (20 pts) Suppose that there are two agents, say agents 1 and 2, and two commodities, say x^1 and x^2. Let u_1 be the preference of agent 1 and u_2 the preference of agent 2.

1. (6 pts) Provide the definitions of weak and strong Pareto efficiency. Under what assumptions on u_1 and u_2, these two concepts are equivalent.
2. (8 pts) Provide the definition of the Walrasian equilibrium. Assume that $u_1(x_1^1, x_1^2) = (x_1^1)^a (x_1^2)^{1-a}$, and that $u_2(x_2^1, x_2^2) = (x_2^1)^b (x_2^2)^{1-b}$, where x_1^1 is the amount of commodity 1 consumed by agent 1, x_1^2 the amount of commodity 2 consumed by agent 1, x_2^1 the amount of commodity 1 consumed by agent 2, and x_2^2 the amount of commodity 2 consumed by agent 2. Of course, a and b are real numbers between 0 and 1. Furthermore, assume that the endowment of agent 1 is $w_1^1 = 1$ and $w_1^2 = 0$, and that the endowment of agent 2 is $w_2^1 = 0$ and $w_2^2 = 1$. Given these assumptions, calculate the Walrasian equilibrium.
3. (6 pts) Assume that $x^1 = 1$ and $x^2 = 2$. Provided $u_1(x_1^1, x_1^2) = \min\{x_1^1, 2x_1^2\}$ and $u_2(x_2^1, x_2^2) = \min\{x_2^1, 2x_2^2\}$, describe all weakly Pareto-efficient allocations in the Edgeworth box. Do the same thing for the situation where $u_1(x_1^1, x_1^2) = x_1^1 + x_1^2$ and $u_2(x_2^1, x_2^2) = \max\{2x_2^1, x_2^2\}$.