

# 國立中央大學九十學年度碩士班研究生入學試題卷

所別: 資訊管理學系 丙組 科目: 微積分 共 / 頁 第 / 頁

1. Find the direction of most rapid increase of  $f(x, y, z) = x^2 + yz$  at  $(1, 1, 1)$  and give the rate of increase in this direction. (10%)
2. Find the volume under the cone  $z = 3\sqrt{x^2 + y^2}$  and over the circle  $x^2 + y^2 \leq a^2$ . (10%)
3. Suppose  $a > 0$ ,  $b > 0$ , and  $c > 0$ . Find the min of  $\frac{a^2}{x} + \frac{b^2}{y}$  on  $x + y = c$ . (10%)
4. Find the sum of the following series. (10%)

$$\frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \frac{x^{10}}{5!} + \dots$$

5. Evaluate  $\iint y \, dx \, dy$  over  $a \leq x \leq y \leq b$ . (10%)
6. Suppose \$100 is invested at 8% compounded  $n$  times per year.
  - (a) Give the equation of the balance  $b(n)$  at the end of 1 year. (5%)
  - (b) Calculate  $\lim_{n \rightarrow \infty} b(n)$ . (5%)
7. A company's annual sales for the first 3 years are shown in the following table.

Year	1	2	3
Sales	2	6	4

- Derive the equation of the least squares line (regression line). (10%)
8. Find the maximum and minimum values of  $f(x, y) = xy$  subject to  $x^2 + y^2 = 8$ . (10%)
9. Suppose the value of a stock on day  $t$  is given by
 
$$\frac{dv}{dt} = k(L - v(t)).$$
 Find the general solution for  $v(t)$ . (10%)
10. Use the Taylor series at 0 for  $f(x) = e^{-x}$  to approximate  $e^{-0.1}$  with an error of no more than 0.0005. (10%)