國立中央大學98學年度碩士班考試入學試題卷

所別：資訊工程學系碩士班　科目：離散數學與線性代數　共 4 頁 第 1 頁

所別：軟體工程研究所碩士班

＊請在試卷答案卷（卡）內作答

1～5 單一選擇題 (每題答對給 3 分、答錯扣 3 分、不答 0 分)

1. Every $n \times n$ matrix has n eigenvalues and n eigenvectors. (A) True. (B) False.
2. Similar matrices have the same (A) solution set. (B) general structure. (C) eigenvalues. (D) eigenvectors.
3. If A is row equivalent to the identity matrix **I**, then A is diagonalizable. (A) True. (B) False.
4. If A is a square matrix with orthonormal columns, then A is invertible. (A) True. (B) False.
5. If $n \times n$ matrix A has n linearly independent eigenvectors, then the linear system $A x = b$ has a least-squares solution $R^{-1} Q^{T} b$. (A) True. (B) False.

6～22 多重選擇題 (每題答對給 5 分、答錯扣 2 分、不答 0 分)

6. Let $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ be the matrix to represent a binary relation **R** on a four-elements set. Which of the following statements are true?

a) There is only 1 connected component in a directed graph that represents **R**.
b) **R** is a partial ordering relation.
c) **R** is reflexive.
d) **R** is an equivalence relation.
e) none of the above.

7. Consider the same relation **R** in question 6. How many 1s are there in the matrix that represents the transitive closure of **R** on **A**?

a) 5　b) 10　c) 9　d)16　e)none of the above.

8. Suppose x and y are integer numbers, and we define the following predicates:

$$Q(x, y) : x = y; E(x): \text{x is even}; O(x): \text{x is odd}$$

Which of the following clauses are correct interpretations of the logical statement:

$$\forall x, y, O(x) \wedge E(y) \rightarrow \neg Q(x, y)$$

a) No odd integer can equal to any even integer".
b) If two integers are not equal, they are one odd and one even integers.
c) If two integers are equal, they can not be one odd and one even integers.
d) Some odd integer is not equal to an even integer.
e) none of the above.

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9. To analyze the complexity of the following procedure P, We will use the following assumptions: Suppose P and B are both procedures. B take $\theta(m)$ time to compute, where m is the size of input; each statement line in and outside the loop counts 1 step.

Procedure P(array1[$a_1, a_2, \ldots, a_n$])
1. if n<3 exit.
2. call B(array1[$a_1, a_2, \ldots, a_n$])
declare initially new empty array2, array3;
3. for (i=1 to n)
4. if ((i mod 2) =0)
5. insert a_i into array2
else
6. insert a_i into array3;
//end for-loop
7. call P(array2);
8. call P(array3);

Suppose n is a multiple of 2, which of the following relations on C_n can describe the complexity of procedure P with respect to problem size n?

a) $C_n = 2C_{n-2} + \theta(n)$ b) $C_n = 2C_{n/2} + C_n + \theta(n)$ c) $C_n = 2C_{n/2} + \frac{3}{2}\theta(n)$
d) $C_n = 2C_{n/2} + \theta(n) + \theta(1)$ e)none of the above

10. What is the time complexity level of the procedure P in question 9?

a) $O(n^{\sqrt{2}})$ b) $O(n \log n)$ c) $O(n^{\log n})$ d) $O(n^2)$ e) none of the above

11. Suppose the population of a cat community is fast growing, each year the size of the population will be surveyed. On the *i-th* year, the population of the *(i-1)th* year will give birth to new baby cats, and the size of new baby cats is the same size of all the population of *(i-1)th* year. There will be extra $2^i + i$ cats moving into this community on the *i-th* year. Suppose no cats missing or dead during the observation years. What of the following function P_n shows the population size of *n-th* year?

a) $P_n = P_{n-1} + 2^n + n$ b) $P_n = 2P_{n-1} + 2^n + n$ c) $P_n = 2P_{n-1} + 2^{n-1} + n - 1$
d) $P_n = P_{n-1} + P_{n-2} + 2^n + n$ e) none of the above.

12. When finding the generating function *f(x)* for P_n in question 11, which of the following is true if the *0-th* year has only one cat?

a) $f(x) - 2f(x) - (x/(1-x)^2) + (1/(1-x)) = 0$
b) $f(x) - 2xf(x) - (x/(1-x)^2) - (1/(1-2x)) = 0$
c) $f(x) - 2xf(x) - (x/(1-2x)^2) + (1/(1-x)) = 0$
d) $f(x) = (2/(1-2x)^2) + (1/(1-2x)) - (1/(1-x)^2) - (1/(1-x))$
e) $f(x) = (1/(1-2x)^2) + (2/(1-2x)) - (1/(1-x)^2) - (2/(1-x))$

13. What are correct solution for P_n in question 11 and 12?

a) $n \times 2^n - 3n - 3$ b) $(n \times 2^n) + 2^{n+1} - 3n - 3$ c) $(n+3) \times 2^n - 3n - 1$

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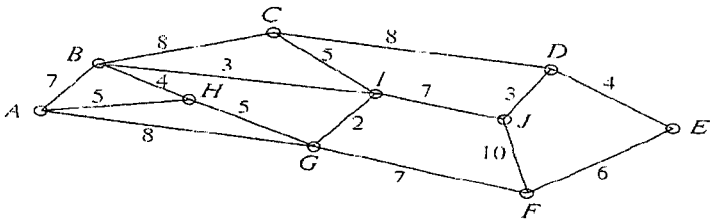
d) $n\times 2^n+3\times 2^n-n-3$　e) none of the above

14. What is the length of the shortest path from node A to D in the following graph.



a)20　b)23　c)25　d)28　e)none of the above

15. Suppose a set S has 4 elements. Consider the subset relation $\subseteq$ among all possible subsets of S. Which of the following statements are true?

a) It is an equivalence relation.

b) It is a partial ordering relation..

c) It is a total ordering relation.

d) It is a lattice.

e) If each possible subset is a node, an edge from node i to node j existed if $i \subseteq j$. The graph will have 48 edges. (an edge from node i to node i counts only one)

16. Let $\mathbf{A}=\begin{bmatrix} 1 & 4 & 5 & 6 \\ 3 & -2 & 1 & 4 \\ -1 & 0 & -1 & -2 \\ 2 & 3 & 5 & 7 \end{bmatrix}$.

Find the rank and nullity of matrix **A**. (a) rank=2, nullity=2 (b) rank=3, nullity=1 (c) rank=2, nullity=4 (d) rank=1, nullity=3.

17. Let **A** be an $m\times n$ matrices, **P** be an invertible $m\times m$ matrix, and **Q** be an $n\times n$ matrix. Which of the following statements are true?

(a) rank(**AQ**)=nullity(**A**) (b) rank(**AQ**)=rank(**A**) (c) rank(**PA**)=rank(**P**)
(d) rank(**PA**)=rank(**A**)

18. Let $\mathbf{A}=\begin{bmatrix} \lambda-5 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 2 \\ 3 & 1 & \lambda-1 & 3 \\ 2 & 4 & 3 & 5 \end{bmatrix}$.

Find all values of λ for which matrix **A** is not invertible. (a) λ=5 (b) λ=5/7 (c) λ=3 (d) λ=3/14.

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19. Let $B = \left\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}\right\}$ and $\hat{B} = \left\{\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\right\}$. Suppose that $[T]_B = \begin{bmatrix}4 & -2\\2 & 3\end{bmatrix}$ is the linear transformation relative to the basis B. Then the corresponding matrix $[T]_{\hat{B}}$ relative to the basis $\hat{B}$ is (a) $\begin{bmatrix}1 & -2\\5 & 6\end{bmatrix}$ (b) $\begin{bmatrix}6 & 4\\-1 & 2\end{bmatrix}$ (c) $\begin{bmatrix}-2 & 3\\6 & 1\end{bmatrix}$ (d) $\begin{bmatrix}6 & -1\\-4 & 2\end{bmatrix}$

20. Suppose T: $\boldsymbol{R}^2 \to \boldsymbol{R}^2$ is a linear transformation and that $T(1,0)=(2,4)$, $T(2,1)=(3,7)$. Which of the following statements are true? (a) T is one to one (b) T is onto (c) $T(3,1)=(5, 11)$ (d) rank(T)=2.

21. If $\boldsymbol{A}$ is an $n \times n$ symmetric matrix, then (A) $\boldsymbol{A}$ is always diagonalizable. (B) $\boldsymbol{A}$ is orthogonally diagonalizable. (C) $\boldsymbol{A}$ has nonnegative eigenvalues. (D) $\boldsymbol{A}$ always has n linearly independent eigenvectors. (E) $\boldsymbol{A}^2$ is also symmetric.

22. Find a singular value decomposition $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathrm{T}}$ with $\boldsymbol{U}$ and $\boldsymbol{V}$ being both orthogonal matrices, where $\boldsymbol{A} = \begin{bmatrix}0 & 1 & 1\\1 & 1 & 0\end{bmatrix}$. Which values are in $\boldsymbol{U}$ or $\boldsymbol{V}$ matrices, (A) $1/\sqrt{2}$. (B) $1/\sqrt{3}$. (C) $1/\sqrt{5}$. (D) $1/\sqrt{6}$. (E) $1/3$.

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