立中央大學八十三學年度研究所碩士班入學試題卷

所別： 資訊工程研究所 組 科目： 線性代數 共 1 頁 第 1 頁

1. (15%) Give a linear system $Ax = b$, $\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$. Let $A_{cof} = \begin{bmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{bmatrix}$,

where A_{ij} is the cofactor of a_{ij}.

(a) (5%) What is the cofactor A_{ij} of a_{ij}?

(b) (5%) Show that $A^{-1} = (1/det\ A)\ A_{cof}$.

(c) (5%) Show that $x_j = det\ B_j / det\ A$, where B_j is the matrix obtained from A by replacing the jth column with the vector b (Cramer's rule).

2. (14%) Let matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

(a) (10%) Factor A into $S\Lambda S^{-1}$ to find S and Λ, where S is the eigenvector matrix and Λ is the eigenvalue matrix of A.

(b) (4%) Find A^{69}.

3. (21%) True or False (Give a reason if true, and give a counterexample if false).

Let A and B be two different $n \times n$ nonsingular matrices.

(a) (3%) $det\ A^T = det\ A$.

(b) (3%) $det\ (A+B) = det\ A + det\ B$.

(c) (3%) AB and BA have the same eigenvalues.

(d) (3%) If A has n different eigenvectors, then A has n independent eigenvectors.

(e) (3%) If A has repeated eigenvalues, then A may be diagonalizable and invertible.

(f) (3%) If A has zero eigenvalue, then A may be diagonalizable and invertible.

(g) (3%) Triangular factor $A = LDU$, where L and U have 1's on the diagonal, and D is a diagonal matrix. {eigenvalues of A} = {eigenvalues of D}.

4. (15%) In the vector space $\mathbf{R}^3$, what is the axis of rotation, and the angle of rotation, of the transformation that takes vector $(x_1,x_2,x_3)^T$ into vector $(x_2,x_3,x_1)^T$? Find the matrix that represents this transformation.

5. (20%) Let S be the subspace of $\mathbf{R}^4$ containing all vectors $(x_1,x_2,x_3,x_4)^T$ with $x_1+x_2+x_3+x_4 = 0$ and $x_1+2x_2+3x_3+4x_4 = 0$.

(a) (10%) Find two bases for the space S and the space $S^\perp$ (the space containing all vectors orthogonal to S) respectively.

(b) (10%) Find the projection of $(1,2,7)^T$ onto the space $S^\perp$.

6. (15%) Let A and B be two $n \times n$ square matrices.

(a) (8%) Show that rank(AB) is less than or equal to the minimum of rank(A) and rank(B).

(b) (7%) Use the result of (a) to show that if AB is invertible then both A and B are invertible.