1. Let A be a set containing four elements. (A) How many reflexive relations are there on A ? (B) How many equiva-lence relations are there on A ? (20%)

2. The set B of balanced parenthesis strings is recursive-ly defined as follows: (1) The empty string is in B . (2) If x is in B , so is (x) . (3) If x and y are in B , so is xy . The problems are: (A) How many balanced parenthesis strings in B are of length 2n ? (B) Can you give a combinatorial interpretation of your answer? (20%)

3. Discuss for what values of r, m, and n is the complete tripartite graph $K_{r,m,n}$ planar? (20%)

4. In a weighted directed graph G = (V,E,C), let d(j) denote the length of some directed path from the source vertex s to vertex j . Show that the numbers d(j) repre-sents the shortest path distance from vertex s to vertex j if and only if

$$d(j) \leq d(i) + c(i,j) \quad \text{for all edge } (i,j) \text{ in } E .$$ (20%)

5. Among n persons, a celebrity is defined as someone who is known by everyone but does not know anyone. The problem is to identify the celebrity if one exists, by asking questions only of the form, "Excuse me, do you know the person over there?" Design an algorithm to solve this problem so that the number of questions is minimized. (20%)