科目：工程數學 C(5005)

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參考用

一、 (5%) EM wave propagates inside an absorptive material. The absorbed intensity amount per penetration depth is proportional to the intensity at that position. Write down a mathematic model to describe the phenomenon above and obtain the general solution.

二、 (一) (5%) Solve the initial value problem, $xy' = y + \sqrt{x^2 + y^2}$, $y(2) = 0$.

(二) (5%) Solve the initial value problem, $\begin{array}{l} y_1' - y_1 - y_2 = 3x \\ y_1' + y_2' - 5y_1 - 2y_2 = 5 \end{array}$, $y_1(0)=3$, $y_2(0)=4$.

三、(10%) The differential equation: $y''(t) + ay'(t) + by(t) = u(t)$, where a and b are constants and $u(t)$ is the unit step function. All initial conditions are zero.

(一) (5%) Solve $y(t)$ when $a = 2$ and $b = 4$.

(二) (5%) Solve $y(t)$ when $a = 4$ and $b = 4$.

四、(5%) Let $x(t) = \cos(t)u(t)$ where $u(t)$ denotes the unit step (or Heaviside step) function. Find the concatenated convolution $x(t) * u(t) * u(t\text{-}1)$ using Laplace transform method, where $*$ is the convolution operator.

五、(15%) Let $y = y(x)$ be a real function of x and consider the following second order differential equation: $x^2 y'' + (6x + x^2)y' + xy = x^2 + 2x$

Find the general Frobenius series solution of y.

注意：背面有試題

六、(5%) Let $f(t) = \sin(\pi t)$ for $t \in (-\pi, \pi]$ be a function of period 2π. Find the Fourier series representation of $f(t)$.

七、(15%) Mark each statement "True" or "False". Just write down your answers. There is no need to specify reasons.

(一) If the columns of $\mathbf{A}$ are linearly independent, then $\mathbf{Ax} = \mathbf{b}$ has exactly one solution for every $\mathbf{b}$.

(二) If U and W are two subspaces of a vector space V, the intersection of U and W is also a subspace of V.

(三) A square matrix with distinct eigenvalues is diagonalizable.

(四) If two square matrices have the same determinant, then they are similar.

(五) If T is a linear transformation and $\{\mathbf{u_1} \cdots \mathbf{u_k}\}$ is a linearly independent set in the domain of T, then $\{T(\mathbf{u_1}) \cdots T(\mathbf{u_k})\}$ is also linearly independent.

八、(10 %) Consider the two sets of linear equations

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \text{(E1)}$$

and

$$\begin{bmatrix} 2 & 0 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad \text{(E2)}$$

(一) (4%) For the inconsistent set, find the least squares solution.

(二) (6 %) For the consistent set, find the real-valued solution with the minimal two-norm (the two-norm of a vector $\mathbf{w} = [w_1 \cdots w_n]^T \in \mathbf{R}^n$ is defined to be $\|\mathbf{w}\|_2 = \sqrt{w_1^2 + \cdots + w_n^2}$).

注意：背面有試題

九、(10%) This problem set discusses how to solve one-dimensional wave equation by Fourier transform. Wave equation: $\frac{\partial^2 u(x,t)}{\partial t^2} - c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = 0$, where c is a constant, $-\infty < x < \infty$, and $0 < t < \infty$. No physical boundary. Initial displacement: $u(x,0) = f(x)$, initial velocity: $\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = 0$.

(一) (2%) Which independent variable (x or t) should be selected for Fourier transform? Why?

(二) (2%) Perform Fourier transform for the wave equation to derive an ordinary differential equation. Use the notation of either $F_x\{u(x,t)\} = U(\xi,t)$ or $F_t\{u(x,t)\} = U(x,\omega)$ when variable x or t is selected.

(三) (3%) Solve the ordinary differential equation derived in (二) with suitable boundary or initial conditions.

(四) (3%) Perform inverse Fourier transform for the solution in (三) to derive the final solution $u(x,t)$. Represent your result by parameters f, x, t, c only.

參考用

十、(15%) Consider a two-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \qquad 0 \le x \le 2, 0 \le y \le 1$$

with the following boundary and initial conditions

$$u(0,y,t) = u(2,y,t) = 0$$

$$\frac{\partial u}{\partial y}(x,0,t) = \frac{\partial u}{\partial y}(x,1,t) = 0$$

$$u(x,y,0) = 0$$

$$\frac{\partial u}{\partial t}(x,y,0) = 1$$

(一) (5%) Derive the eigenvalues and the corresponding eigenfunctions.

(二) (5%) What is lowest frequency in the motion of the solution (the fundamental frequency)?

(三) (5%) Solve $u(x,y,t)$.