(1) Based on the following Schrodinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi$$, Prove that probability density $\rho = \int_{-\infty}^{\infty}|\psi(x,t)|^2 dx$ is a

time-independent constant if ψ satisfies the condition of square-integrable. (20%)

(2) Simple harmonic oscillator system can be described by the Schrodinger

equation $\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{2}m\omega^2 x^2\psi = E\psi$, prove that its Hamiltonian can well be

written as $H = \hbar\omega(a^+a + \frac{1}{2})$, where $a = \frac{1}{\sqrt{2\hbar m\omega}}(ip + m\omega x)$ and

$a^+ = \frac{1}{\sqrt{2\hbar m\omega}}(-ip + m\omega x)$. Note p and x denote, respectively, the momentum and

position. (20%)

(3) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$. Meanwhile,

prove that these eigenvectors are orthogonal. (20%)

(4) Density of states $D(\varepsilon)$ means the number of eigenstates in a small interval dε
around energy ε. According to such a definition. Describe $D(\varepsilon)$ as a function of
ε for different semiconductor heterostructures: quantum wells, wires and dots.
(20%)

(5) Write out the Fermi-Dirac distribution function and the Boson-Einstein
distribution function. (10%)

(6) Briefly describe what the Bloch theorem is. (10 %)