(0,K)

國立中央大學94學年度碩士班考試入學試題卷

所別：**電機工程學系碩士班 乙組** 科目：**近代物理**

1. Use Schrodinger's time-dependent wave equation to derive the continuity equation $\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$, where $\rho(\vec{x},t)$ stands for $|\psi(\vec{x},t)|^2$ and $\vec{J}(\vec{x},t) = \frac{\hbar}{m}\mathrm{Im}(\psi^* \nabla \psi)$. (15%)

2. Use uncertainty principle to calculate the ground-state energy of one dimensional simple harmonic oscillator described by the Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$. (15%)

3. Calculate the ground-state energy of system, which is described by the Hamiltonian $H = \frac{-\hbar^2 \partial^2}{2m\partial x^2} + V(x)$, where $V(x) = \begin{cases} \infty, & \text{if } x \le 0 \\ \frac{1}{2}m\omega^2 x^2, & \text{otherwise} \end{cases}$. (15 %)

4. The Hamiltonian for a particle of mass m and charge q in a magnetic field $\boldsymbol{B}$ is $H=(\mathbf{p}-q\mathbf{A}/c)^2/2m$. If $\mathbf{A}$=B/2 (-y **i**+ x **j**), and then magnetic field is constant and along the z axis. Assume B is small and drop the quadratic in B. Prove that the interaction Hamiltonian is $H_{\text{int}} = -\frac{q}{2mc}\boldsymbol{L} \bullet \boldsymbol{B} = \boldsymbol{\mu} \bullet \boldsymbol{B}$, where $\boldsymbol{L}$ and $\boldsymbol{\mu}$ are the angular momentum and magnetic moment. (10%)

5. Consider the following Hamiltonian
$$H = H_0 + H_1 = \frac{p^2}{2m} + V(x) - qfx$$
, where $V(x) = \begin{cases} 0, & \text{for } |x| \le \frac{L}{2} \\ \infty, & \text{otherwise} \end{cases}$ and $H_1 = -qfx$. q and f denote, respectively, the charge and strength of electric field. Note that q is positive. Discuss the effect of perturbed term H_I on the energy levels to the second order using time-independent perturbation theory. (25%)

6. List similarities and differences between phonons and photons.(10%)

7. Make a rough estimate of the binding energy of the donor electron of arsenic in a germanium crystal, taking the dielectric constant of the crystal to have the value $\kappa = 16$ and the effective mass of the electron to have the value $m^* = 0.2m$. (10 %)