

所別：電機工程學系碩士班 甲組(一般生) 科目：工程數學  
甲組(學位在職生)  
乙組(一般生)、(學位在職生)  
丙組(一般生)、丁組

1. Find a basis of solutions of the following differential equation using the Frobenius method (Show the details of your work) (15%)

$$x^2 y'' + 6xy' + (6 - 4x^2)y = 0$$

2. Find the Laplace transform of a piecewise continuous periodic function  $f(t)$  with period  $p > 0$  (i.e.,  $f(t+p) = f(t)$  for all  $t > 0$ ). (Show the details of your work) (15%)

3. Prove that the eigenvectors of matrix  $A = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$  are orthogonal. (10%)

4. The eigenvalues of matrix A are  $\lambda_1 = e^{i\theta}$  and  $\lambda_2 = e^{-i\theta}$  corresponding, respectively, to the eigenvectors  $X_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$  and  $X_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$ . Find the elements of matrix A. (20%)

5. Show that the given integral represents the indicated function. (10%)

$$\int_0^{\infty} \frac{\sin \omega}{\omega} \cos x\omega d\omega = \begin{cases} \pi/2 & \text{if } 0 \leq x < 1 \\ \pi/4 & \text{if } x = 1 \\ 0 & \text{if } x > 1 \end{cases}$$

6. Find all the solutions to the equation:  $\cos z = 20$ . (15%)

7. Evaluate  $\oint_C \frac{\sin 2z}{(z-i)^3} dz$ , with  $C: |z-1| = 2$ . (15%)