國立中央大學八十八學年度轉學生入學試題卷

大氣科學系　三年級　科目：應用數學　共 1 頁 第 1 頁

1. Solve the following differential equation :

$$(xy - y^2)dx - x^2 dy = 0$$

(10%)

2. Solve the following differential equation :

$$xy' + (1+x)y = e^x$$

(10%)

3. Let

$$A = \begin{bmatrix} 5 & -1 & 2 \\ 1 & 2 & -3 \\ -2 & 0 & 1 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 0 & -1 & 3 \\ 5 & 0 & 4 \\ 1 & -3 & 0 \end{bmatrix}$$

Find the following expressions :

(a) $A + A^T + B - B^T$ (10%)

(b) AB

4. Find the Laplace transform of the given function f(t)

$$f(t) = \begin{cases} t & \text{if } 0 < t < \pi \\ t - \pi & \text{if } \pi < t < 2\pi \end{cases}$$

$$f(t) = f(t + 2\pi)$$

(10%)

5. Evaluate the surface integral

$$\iint_S \vec{F} \cdot \hat{n} dA$$

where $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$

and S the surface of the cube

$$\begin{cases} 0 \le x \le a \\ 0 \le y \le b \\ 0 \le z \le c \end{cases}$$

(15%)

6. Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{dx}{x^2 + 1} \qquad x : \text{real}$$

(15%)

7. Solve the equation for z

$$\ln z = 2 + \frac{1}{4}\pi i$$

(15%)

8. Find solution u(x,t) by separating variables, if

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2} \quad ;$$

$$u(0,t) = 0 \quad ;$$

$$u(10,t) = 0 \quad ;$$

$$u(x,0) = 5\sin 2\pi x \quad ;$$

(15%)

參考用