國立中央大學八十九學年度轉學生入學試題卷

財務管理學系　二年級　科目：微積分　共一頁　第一頁

參考用

1 (15 points) Evaluate the given integrals.

(a) $\int \cos(\ln x)dx$. (b) $\int_0^2 \int_y^2 e^{x^2} dxdy$. (c) $\int_{-1}^2 \frac{x}{\sqrt{x+2}} dx$.

2 (15 points) Find $\frac{dy}{dx}$ if

(a) $\ln \frac{y}{x} = x^2 y^3$. (b) $y = x^{\sqrt{x}}$. (c) $y = \frac{e^{-3x}\sqrt{2x-5}}{(6-5x)^4}$.

3 (10 points) When the price of a certain commodity is p dollars per unit, the manufacturer is willing to supply x thousand units, where

$$x^2 - 2x\sqrt{p} - p^2 = 31.$$

How fast is the supply changing when the price is \$9 per unit and is increasing at the rate of 20 cents per week?

4 (10 points) Find the indicated limits.

(a) $\lim_{x\to\infty} e^{-x} \ln x$.

(b) $\lim_{x\to\infty} (\sqrt{x} - 1)^{\frac{1}{\sqrt{x}}}$.

5 (10 points) Determine where the given function is increasing, decreasing, concave upward, and concave downward. Find the relative extrema, inflection points and asymptotes (if any) and draw the curve.

$$f(x) = x^2 e^{-x}.$$

6 (10 points) Find the extreme values and the saddle points (if any) of

$$f(x, y) = 2x^3 - 24xy + 16y^3.$$

7 (10 points) Find the area of the region bounded by the curve $y = \frac{1}{x^2}$ and the lines $y = x$ and $y = \frac{x}{8}$.

8 (10 points) Decide if the following improper integrals convergence or divergence. Explain your answer.

(a) $\int_2^\infty \frac{dx}{\sqrt{x^3+1}}$.

(b) $\int_{-\infty}^\infty x^3 dx$.

9 (10 points) Find the volume under the surface $z = e^{-x}e^{-y}$ and above the triangle with vertices (0,0), (1,0), and (0,1).