國立中央大學八十七學年度轉學生入學試題卷

1. You are visiting an island containing two types of people: **knights** who always tell the truth and **knaves** who always lie.

   (a). Two natives A and B speak to you as follows:

   A says: B is a knight.

   B says: A and I are of opposite type.

   What are A and B?(5%)

   (b). Another two natives C and D approach you but only C speaks.

   C says. Both of us are knaves.

   What are C and D?(5%)

   (c) You then encounter natives E and F.

   E says: F is a knight.

   F says: E is a knight.

   What are E and F?(5%)

   (d) Finally you meet a group of six natives, U, V, W, X, Y, and Z, who speak to you as follows:

   U says: None of us is a knight.

   V says: At least three of us are knights.

   W says: At most three of us are knights.

   X says: Exactly five of us are knights.

   Y says: Exactly two of us are knights.

   Z says: Exactly one of us is a knight.

   Which are knights and which are knaves.(10%)

2. (a) Does there exist a binary relation R on { 1,2,3 } such that **R** is reflexive, transitive, symmetric, and antisymmetric? Justify your answer.(7%)

   (b) Is the union of two equivalence relations always an equivalence relation? Justify your answer.(8%)

   (c) If **a**, **b**, and **c** are odd integers, can $ax^2 + bx + c = 0$ have a rational solution? Justify your answer.(10%)

參考用

注意：背面有試題

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3. $K_{m,n,\ell}$ denotes a complete tripartite graph $G = (V, E)$ such that

(1) $V = V_1 \cup V_2 \cup V_3$, where $|V_1| = m$, $|V_2| = n$, $|V_3| = \ell$, and $V_i \cap V_j = \varnothing$ if $i \neq j$.

(2) There is an edge connecting vertices a and b if and only if $a \in V_i$, $b \in V_j$ and $i \neq j$.

(a) Show that $K_{2,2,2}$ is planar.(7%)

(b) Show that $K_{3,2,1}$ is nonplanar.(8%)

(c) Find the necessary and sufficient condition in terms of m, n, and ℓ such that $K_{m,n,\ell}$ is planar.(10%)

4. Develop a general explicit formula for a nonhomogeneous recurrence relation of the form $a_n = ra_{n-1} + s$, where r, s and a_0 are given constants.

(a) $r = 1$.(10%)

(b) $r \neq 1$.(15%)