國立中央大學八十八學年度轉學生入學試題卷

數學系　三年級　　科目：線性代數　　共 2 頁 第 1 頁

1. (20%) Determine a Jordan canonical form for the matrix A.

$$A=\begin{bmatrix} 2 & 2 & -1 \\ -1 & -1 & 1 \\ -1 & -2 & 2 \end{bmatrix}$$

2. (10%)

(a) Let $A=\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 0 & 3 & 4 \end{bmatrix}$. Find a basis for the null space of A and a basis for the range of A.

(b) Determine the nullity and rank of A.

參考用

3. (30%) Let A be a 3×3 matrix such that $A^2=0$.

(a) Prove that the column space of A is contained in the null space of A.

(b) Use this and the rank-nullity theorem to show that rank of A must be 0 or 1.

(c) Use this to show that all the rows of A must be scalar multiples of a single vector.

4. (20%) Let $A=\begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$. You are given that the characteristic polynomial of A is $p(\lambda)=-(\lambda-1)^2(\lambda+2)$, and an eigenvector of A with respect to $\lambda=-2$ is $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

(a) Find a basis for the eigenspace E_λ for $\lambda=1$.

(b) Show that A is diagonalizable by finding matrices S and D such that $S^{-1}AS=D$ where D is a diagonal matrix or explain that A is not diagonalizable.

注意：背面有試題

5 (20%)

(a) Determine whether the subset of $\mathbb{R}^3$,

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 - 3x_2 + 8x_3 = 0 \right\}$$ is a subspace of $\mathbb{R}^3$.

Explain why.

(b) Calculate the dimension of W where

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 = 2x_3 \right\}.$$ Show your work.