國立中央大學103學年度碩士班考試入學試題卷

所別：生物醫學工程研究所碩士班 生醫訊號與器材組(一般生)　科目：工程數學　共 1 頁 第 1 頁

本科考試可使用計算器，廠牌、功能不拘　＊請在試卷答案卷（卡）內作答

1. [20%] Given the function $f(x)=x_1-x_2+2x_1x_2+2x_1^2+x_2^2$ and $x_0=(1,5)^T$, determine the Newton's direction (to minimize f) at x_0.

2. [15%] Solve the initial value problem : $y'+y\tan x=\sin 2x$, $y(0)=1$.

3. [15%] Let $r(t_1,t_2) = a\cos t_1\, i + a\sin t_1\, j + t_2 k$, please get the partial derivatives with respect to t_1 and t_2.

4. [12%] (a) Solve the following differential equation: $\frac{dy(t)}{dt}=\frac{y(t)-M}{T}$, where T and M are constants and the initial condition is $y(0)=M$. [5%] (b) When a pizza is reheated in an oven, its temperature is measured as 200°C. After remove it from the oven to the room temperature (20°C), three minutes later the temperature is 110°C. How long does it take for the pizza to cool off to 50°C? [7%]

5. [15%] In Linear Algebra:

   (1) Under what conditions on b_1 and b_2 (if any), dose $Ax=b$ have a solution?

$$A=\begin{bmatrix}1 & 2 & 0 & 3\\ 2 & 4 & 0 & 7\end{bmatrix},\quad b=\begin{bmatrix}b_1\\ b_2\end{bmatrix}$$

   (2) Find two vectors in the nullspace of A,

   (3) Find the complete solution to $Ax=b$.

6. [15%] Performing the elimination to A and b, so that A is reduced to an echelon form:

$$[A\quad b]=\begin{bmatrix}1 & 2 & 3 & b_1\\ 4 & 5 & 6 & b_2\\ 7 & 8 & 9 & b_3\end{bmatrix}\rightarrow\begin{bmatrix}1 & 2 & 3 & b_1\\ 0 & -3 & -6 & b_2-4b_1\\ 0 & 0 & 0 & b_3-2b_2+b_1\end{bmatrix}$$

   (1) What is the basis for the null space?

   (2) For b to be in the column space, what condition does b has to meet?

   (3) What is the basis for the left-null space?

   (4) What is the basis for the column space?

7. [8%] How do you use Differential Equations to solve the engineering problems in the real world? Please provide one practical example of using Differential Equations in your personal experience.