所別：統計研究所碩士班 不分組(一般生)　科目：數理統計　共 2 頁 第 1 頁
統計研究所碩士班 不分組(在職生)

本科考試可使用計算器，廠牌、功能不拘　＊請在試卷答案卷（卡）內作答

1. Let X_1, X_2 and X_3 denote a random sample from the distribution having p.d.f. $f(x)=e^{-x}, 0<x<\infty$.
Show that
$$Y_1=\frac{X_1}{X_1+X_2},\quad Y_2=\frac{X_1+X_2}{X_1+X_2+X_3},\quad Y_3=X_1+X_2+X_3$$
are mutually independent. (15%)

2. Let X has the logistic p.d.f. $f(x)=e^{-x}/(1+e^{-x})^2, -\infty<x<\infty$.
(a) Find the distribution function $F(x)$ of X. (5%)
(b) Show that $f(x)=F(x)[1-F(x)]$. (5%)
(c) Show the moment-generating function M(t) of X is $\Gamma(1-t)\Gamma(1+t), -1<t<1$. ($\Gamma(\alpha)=(\alpha-1)!$) (5%)

3. Let random variables X, Y be jointly distributed with p.d.f. given by
$$f(x,y)=\frac{2}{n(n+1)}$$
where $y=1,\ldots,x$; $x=1,\ldots,n$.
Compute
(a) $E(X\,|\,Y=y)$, (5%)
(b) $E(Y\,|\,X=x)$, (5%)
(c) correlation coefficient $\rho(X,Y)$. (5%)

4. Let X_1, X_2 be two random variables with joint m.g.f. given by
$$M(t_1,t_2)=[\frac{1}{3}(e^{t_1+t_2}+1)+\frac{1}{6}(e^{t_1}+e^{t_2})]^2,\quad t_1,t_2\in R.$$
Calculate
(a) $E(X_1)$ (5%)
(b) Var(X_1) (5%)
(c) Cov(X_1, X_2) (5%)

5. Let $X_1,\ldots, X_n$ be a random sample from $N(\mu,\sigma^2)$, where both μ and σ^2 are unknown. Find the UMVUE estimator of μ/σ. (10%)

注意：背面有試題

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6. Let $X_1,\ldots, X_n$ be i.i.d. random variables with p.d.f. given by

$$f(x)=\frac{1}{\theta}e^{-x/\theta}, 0<x<\infty, 0<\theta<\infty.$$

(a) Derive the UMP test for testing the hypothesis
$H_0:\theta\ge\theta_0$ vs. $H_1:\theta<\theta_0$ at level of significance α. (10%)

(b) Determine the minimum sample size n required to obtain power at least 0.95 against the alternative θ_1=500 when $\theta_0=1000$ and α=0.05. (10%)

7. Let $X_1,\ldots, X_n$ be i.i.d. random variables with p.d.f. given by

$$f(x)=\frac{1}{\theta_2}\exp\left[-\frac{(x-\theta_1)}{\theta_2}\right],\ x>\theta_1\ ,\theta_1\in R,\ 0<\theta_2<\infty.$$

Find the maximum likelihood estimates of θ_1 and θ_2. (10%)

參考用

注意：背面有試題