國立中央大學103學年度碩士班考試入學試題卷

所別:機械工程學系碩士班 丁組(系統)(一般生)

科目:工程數學(含程式設計) 共 2 頁 第 1 頁

機械工程學系光機電工程碩士班 甲組(機電系統控制)(一般生)

本科考試可使用計算器, 廠牌、功能不拘

*請在試卷答案卷(卡)內作答

Ordinary Differential Equations

- 1. Find the solution for the following ordinary differential equations (ODEs):
 - (a) y' + y = -x/y (5%)
 - (b) $y'' + 4y' + 3y = 65\cos(2x)$ (5%)

(c)
$$\begin{cases} y_1' = y_1 + y_2 + 10\cos x \\ y_2' = 3y_1 - y_2 - 10\sin x \end{cases}$$
, find y_1 and $y_2 = ?$ (5%)

2. For a homogenous ODE given as:

$$x^{3}y'''-3x^{2}y''+6xy'-6y=0$$
 (1)

- (a) Find three solutions $y_1(x)$, $y_2(x)$, and $y_3(x)$ that can form a basis of solutions, show that they are linear independent, for Eq. (1). (5%)
- (b) If there is a non-homogenous term $r(x) = x^4 \ln x$ of Eq. (1), then Eq. (1) becomes $x^3 y''' 3x^2 y'' + 6xy' 6y = x^4 \ln x$ (2) find the particular solution for Eq. (2), $y_p(x) = ?$ (5%)

Laplace Transform and Fourier Analysis

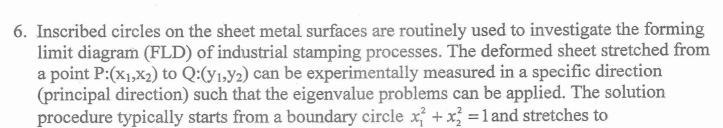
- 3. Solve the integro-differential equation $y'(t) = 1 e^{-2t} \int_0^t y(\tau) e^{2\tau} d\tau$, y(0) = 1 (10%)
- 4. $f(x) = x^2$, $0 < x < 2\pi$, $f(x) = f(x + 2\pi)$
 - (a) Find the Fourier series. (10%)

(b) Evaluate
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = ?$$
 (5%)

Linear Algebra and Vector Analysis

5. Solve the following linear systems

(a)
$$\begin{bmatrix} 2 & 3 & 1 & -11 \\ 5 & -2 & 5 & -4 \\ 1 & -1 & 3 & -3 \\ 3 & 4 & -7 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 3 \\ -7 \end{bmatrix}$$
(8%) (b)
$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$
(7%)



$$\frac{y_1^2}{\lambda_1^2} + \frac{y_2^2}{\lambda_2^2} = 1$$
, where $\lambda_1 \lambda_2$ are eigenvalues. The transformation matrix A is designated

as
$$y = Ax$$
; or $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0.5 & 1.5 \\ 1.5 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ can be determined experimentally. Please solve

eigenvalues and eigenvectors to find the principal directions and indicate the shape of the deformed boundary. (10%)



國立中央大學103學年度碩士班考試入學試題卷

所別:機械工程學系碩士班 丁組(系統)(一般生) 科目:工程數學(含程式設計) 共 2 頁 第 2

機械工程學系光機電工程碩士班 甲組(機電系統控制)(一般生

科考試可使用計算器,廠牌、功能不拘 *請在試卷答案卷(卡)內作答

程式設計

7. (15%) Heron's formula for the area A, of a triangle with sides of length a, b, c is

$$A = \sqrt{[s(s-a)(s-b)(s-c)]},$$

where

$$s = \frac{\left(a+b+c\right)}{2}.$$

White a function that accepts the value of a, b, and c as parameters from a calling function, and then calculates the values of [s(s-a)(s-b)(s-c)]. If this quantity is positive, the function calculate A. If the quantity is negative, a, b, and c do not form a triangle, and the function should set A = -1. The value of A should be returned by the function. The code is limit to C, C++, Visual Basic or Fortran programing language, and please state before your answer. All variables are declared to real numbers.

8. (10%) In sorting techniques, the most familiar algorithm is the bubble sort, in which successive value in the array are compared, beginning with the first two element. If the array is to be sorted in ascending (from smallest to largest) order, the smaller value of the two being compared is always placed before the larger value. The pseudo-code is as follows:

For the first element in the array to 1 less than the last element (i index)

For the second element in the array to the last element (j index)

If num[j] < num[j-1] then

Swap num[j] with num[j-1]

End if

End For

End For

Write a program to implement the bubble sort. The code is limit to C, C++, Visual Basic or Fortran programing language, and please state before your answer. All variables are declared to real numbers except for i, j index to the integer.

