甲、計算、證明題：共 2 題，每題 10 分，共 20 分。須詳細寫出計算及證明過程，否則不予計分。

1. The production of Q units of a commodity is related to the amount of labor x and the amount of capital y (in suitable units) expended by the equation $Q = f(x,y) = x^{3/4}y^{1/4}$. If an expenditure of 100 units is available for production, how should it be apportioned between labor and capital so that Q is maximized?

2. A manufacturer of aquariums wants to make a large rectangular box-shaped (an open rectangular box) aquarium (開口的長方體水族箱) that will hold 64 cubic feet of water. If the material for the base costs \$ 20 per square foot and the material for the sides costs \$ 10 per square foot, find the dimension for which the cost of the materials will be the least.

乙、填充題：共 10 題，每題 8 分，共 80 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. For what value of the constant m is $f(x) = \begin{cases} \cos 3x, & x \le 0 \\ mx, & x > 0 \end{cases}$ differentiable at $x = 0$?
Answer : _____

2. Find the limit. $\lim\limits_{x \to -1} \dfrac{x(x^2 - 1)}{|x^2 - 1|}$. Answer : _____

3. Suppose that the first derivative of $y = f(x)$ is $y' = 6(x+1)(x-2)^2$. At what points does the graph of f have a point of inflection? Answer : _____

4. Find the area under the curve $y = \dfrac{\ln x}{x^2}$ from $x = 1$ to $x = \infty$. Answer : _____

5. Suppose that x and y are related by the equation $x = \displaystyle\int_0^y \dfrac{1}{\sqrt{1 + 4t^2}}\, dt$. Find $\dfrac{dy}{dx}$.
Answer : _____

6. Suppose that the edge lengths $x, y,$ and z of a rectangular box are changing at the following rates: $\frac{dx}{dt} = 1$ m/sec, $\frac{dy}{dt} = -2$ m/sec, $\frac{dz}{dt} = 1$ m/sec. Find the rate at which the box's volume is changing at instant when $x = 4$, $y = 3$, and $z = 2$. Answer : _____

7. Evaluate $\displaystyle\iint_R e^{y^2}\, dA$; R is bounded by $x = 0$, $y = 2x$, and $y = 2$. Answer : _____

8. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \dfrac{(-1)^n}{n}(x - 5)^n$.
Answer : _____.

9. Evaluate the integral. $\displaystyle\int_0^1 \dfrac{x}{\sqrt{4 - 5x}}\, dx$. Answer : _____.

10. Find $f'(2)$ if $f(x) = e^{g(x)}$ and $g(x) = \displaystyle\int_2^x \dfrac{t}{1 + t^4}\, dt$. Answer : _____.