科目 線性代數 類組別 902

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*請在答案卷內作答

參考用

Instructions: Do all problems. Show your work. The notation, $\mathcal{R}$, is the set of all real numbers

1. Consider the matrix A given by

$$A = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 3 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

   (a) Let $T(x); \mathcal{R}^4 \to \mathcal{R}^3$ defined as $T(x) = Ax$, where $x = [x_1, x_2, x_3, x_4]^T$. Show that T is a linear transformation. (10pts)

   (b) Find a basis for the null space of T, the dimension of the null space, $dim(N(T))$ and the dimension of the range space, $dim(R(T))$. (10pts)

   (c) Apply the Gram-Schmidt procedure to produce an orthonormal basis of the row space of A. (10pts)

2. Consider the matrix A given by

$$A = \begin{pmatrix} 2 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 3 & 2 \end{pmatrix}$$

   (a) Write down the characteristic polynomial of the matrix A (10pts).

   (b) Find all eigenvalues of A and corresponding vectors. (10pts)

   (c) Find an invertible matrix Q and a diagonal matrix D such that $D = Q^{-1}AQ$. (10 pts)

   (d) Show that the matrices D and A are similar(10pts) .

   (e) Given $b = [1, -4, 3, -1, 1]^T$, use the Gaussian elimination to solve the linear system $Ax = b$, for $x = [x_1, x_2, x_3, x_4, x_5]^T$. (10pts)

   (f) Show that A is invertible and find A^{-1}. (10pts)

   (g) Is the matrix A symmetric positive definite? Justify your answer. (10pts)