參考用

Problem 1 - Ordinary Differential Equations (ODEs) (20%)

(a) Find the solution for $xy'+4y=8x^4, y(1)=2$ (6 %)

(b) Solve $y''+4y'+(\pi^2+4)y=0, y(\frac{1}{2})=1, y'(\frac{1}{2})=-2$ (6%)

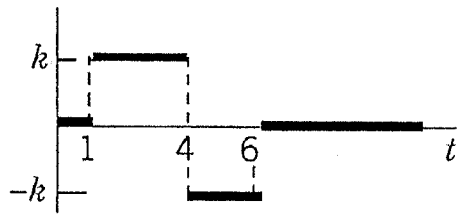
(c) Find the solution to the 3rd-order ODE $x^3y'''+xy'-y=x^2$. (8%)

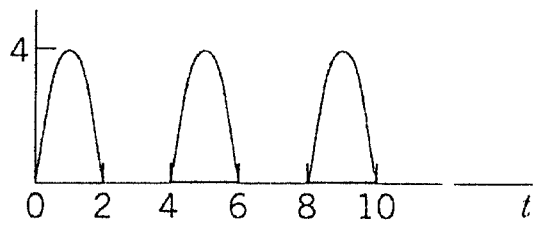
Problem 2 - Power series & Frobenius methods (15%)

For a general **Legendre** equation $(1-x^2)y''-2xy'+n(n+1)y=0$ occurs in models exhibiting spherical symmetry. Please derive the recursion relation. **(5 %)** Please use power series method to solve it as n=1. (10%)

Problem 3 - Laplace transform (25%)

(a) Please write down the unit step function of the following figures. (10%)





(b) Solve $y''+2y'+5y=25t-100\delta(t-\pi); y(0)=-2, y'(0)=5$; δ is the direct delta function. (15%)

Problem 4 - Fourier series (10%)

Let $f(x) = x$ for $-\pi \le x \le \pi$. Please write the Fourier series of f on $[-\pi, \pi]$ and explain the Gibbs phenomenon.

注意:背面有試題

參考用

Problem 5 – Linear Algebra & Vector Calculus (20%)

(a) Find eigenvalues and their corresponding eigenvectors of the matrix

$$\mathbf{C}=\begin{bmatrix}3 & 5 & 3\\ 0 & 4 & 6\\ 0 & 0 & 1\end{bmatrix} \qquad (5\%)$$

(b) For a square matrix **A**, show that $\mathbf{A}^{-1}$ exists if and only if the eigenvalues $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ are all nonzero, and then $\mathbf{A}^{-1}$ has the eigenvalues $1/\lambda_1, 1/\lambda_2, 1/\lambda_3, \ldots, 1/\lambda_n$. (7%)

(c) Find the surface integral of $\int_S \mathbf{F}\cdot\mathbf{n}\,dA$ if $\mathbf{F}=[x^3-y^3, y^3-z^3, z^3-x^3]$, S: the surface of $x^2+y^2+z^2\le 25, z\ge 0$ by the divergence theorem. (8%)

Problem 6 – Complex Analysis (10%)

$z = x + iy$ is a complex number and $f(z) = u(x,y) + iv(x,y)$ is a complex function.

(a) Is the function $f(z)=e^x(\cos y - i\sin y)$ an analytic function? Why? (3 %)

(b) $f(z)=\cos\left[\frac{1}{2}\pi(1+i)\right]$, transfer $f(z)$ in the form of $f(z) = u + iv$ and find its functional value? (3 %)

(c) Find all solutions for the complex function $\cosh z=-1$. (4 %)

注意:背面有試題