國立中央大學104學年度碩士班考試入學試題

所別：太空科學研究所碩士班 不分組(一般生)　科目：應用數學　共 1 頁 第 1 頁
太空科學研究所碩士班 不分組(在職生)

本科考試禁用計算器　＊請在答案卷（卡）內作答

參考用

請注意：作答時，請寫出推導計算步驟或用文字說明如何獲得答案。如果只列出最後答案，卻沒有推導計算步驟或文字說明，該題將不予計分。

(1, 20%) Solve the initial value problem and show the details of your work.

(a) $y' = \dfrac{\tan y}{x-1},\ y(0) = \dfrac{\pi}{2}$ (10%),

(b) $y'' - 2y' = 6e^{2x} - 4e^{-2x},\ y(0) = -1,\ y'(0) = 6$ (10%).

(2, 20%) Determine the type of stability of the critical point (5%). Then find a real general solution (10%) and sketch or graph some of the trajectories in the phase plane (5%).

$$y_1' = -y_1 + 4y_2$$
$$y_2' = 3y_1 - 2y_2$$

(3, 10%) Verify if the Bessel function of the first kind of order n, $J_n(x)$, and $J_{-n}(x)$ are linearly dependent or independent (5%)? What is a general solution of Bessel's equation for all values of n (5%)? It is noted that n is an integer number, $J_n(x) = \lim_{\nu \to n} J_\nu(x)$, and

$$J_\nu(x) = x^\nu \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+\nu} m!\,\Gamma(\nu+m+1)}.$$

(4, 10%) Show the solution of the initial value problem by the Laplace transform.

$$y'' + 16y = 4\delta(t - 3\pi),\ y(0) = 2,\ y'(0) = 0.$$

(5, 20%) Given matrix $\mathbf{M} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ and column vector $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Let $\mathbf{u} = \mathbf{M}^{101}\mathbf{v}$. Determine the column vector $\mathbf{u}$.

(6, 10%) Suppose that $f(x)$ and $g(x)$ are piecewise continuous, bounded, and absolutely integrable on the x-axis. Then

$$\mathcal{F}\{f(x) * g(x)\} = \sqrt{2\pi}\,\mathcal{F}\{f(x)\}\,\mathcal{F}\{g(x)\}.$$

It is noted $\mathcal{F}\{\ \}$ is Fourier transform and $*$ means convolution.

(7, 10%) Is the given series, $\sum_{n=1}^{\infty} \dfrac{(3i)^n n!}{n^n}$, convergent or divergent (5%)? Give a reason and show details (5%).