國立中央大學104學年度碩士班考試入學試題

所別：機械工程學系碩士班 甲組(固力與設計)(一般生)　科目：工程數學　共 2 頁 第 1 頁
機械工程學系碩士班 乙組(製造與材料)(一般生)
機械工程學系碩士班 丙組(熱流)(一般生)
能源工程研究所碩士班 不分組(一般生)
機械工程學系光機電工程碩士班 乙組(光機)(一般生)

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷（卡）內作答

1. Solve $y''+2y=\begin{cases}1 & 0<t<2\\ 0 & t>2\end{cases}$, $y(0)=y'(0)=0$ by Laplace transform, and using unit step functions. Please leave the unit step functions in the answer. (10%)

2. If $r(x)=x^2$, $0<x<2\pi$, $r(x)=r(x+2\pi)$

(a) Find $r(x)$ in the Fourier series (6%)

(b) Evaluate $\sum_{n=1}^{\infty}\frac{1}{n^2}=?$ (3%)

(c) Evaluate $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}=?$ (3%)

(d) Evaluate $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=?$ (3%)

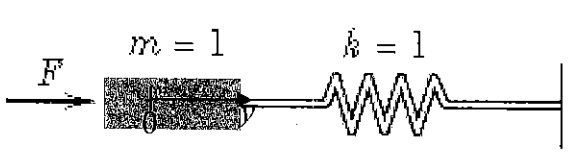
3. Find the solution for the following ordinary differential equations (ODEs):

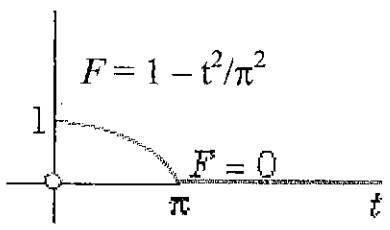
(a) $y'+xy=xy^{-1}$, $y(0)=3$. (5%)

(b) $2x\tan y dx+\sec^2 y dy=0$ (5%)

(c) $x^2y''-3xy'+3y=3\ln x-4$, $y(1)=0$, $y'(0)=1$ (5%)

4. Referring to the following figure, find the equation that can describes (models) the motion of the mass as a function of time, $y(t)$, and solve the modeling equation if the initial position and velocity of the mass are both zeros, i.e., $y(0)=0$ and $y'(0)=0$. (10%)





注意：背面有試題

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5. There are nine components of stress states acting on an infinitesimal element as denoted in (a) and can be transformed into the axes of principal stress as (b).

(a) $\begin{bmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$ (b) $\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$ (c) $\begin{bmatrix} 70 & 20 & 0 \\ 20 & 35 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (d) $\begin{cases} I_1 = 105 \\ I_2 = -2050 \\ I_3 = 170700 \end{cases}$

The magnitudes of principal stress, σ_p, are the roots of $\sigma_p^3 - I_1\sigma_p^2 - I_2\sigma_p - I_3 = 0$. Using the concept of ***eigenvalues***, please deduce $I_1 = \sigma_1 + \sigma_2 + \sigma_3, I_2 = -(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)$, $I_3 = \sigma_1\sigma_2\sigma_3$ (5%). Furthermore, please find the principal stress for the stress state of (c) and (d), respectively (10%)

6. Please find the work required to move a particle by force $F(x^2\hat{i} - yz\hat{j} + x\cos(z)\hat{k})$ along the path $C(x = t^2, y = t, z = \pi t; 0 \le t \le 3)$ from point Q(9,3,3π) to point P(0,0,0) by calculating the integral $-\int_C FdR = -\int_0^3 F_x dx + F_y dy + F_z dz$ (10%)

7. Use separation of variables to find the solution.(10%)

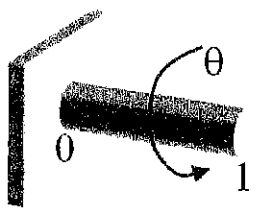
$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} + u$$

8. The twist angle of a torsionally vibrating shaft (see figure) of unit length is determined from:

$$a^2 \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 \theta}{\partial t^2}, 0 < x < 1, t > 0$$

$\theta(0, t) = 0$ $\left.\frac{\partial \theta}{\partial x}\right|_{x=1} = 0, t > 0$

$\theta(x,0) = x$ $\left.\frac{\partial \theta}{\partial t}\right|_{t=0} = 0$ $0<x<1$



The boundary condition at x=1 is called a free-end condition. Solve for θ(x,y) (15%)