國立中央大學104學年度碩士班考試入學試題

所別：財務金融學系碩士班 乙組(一般生)　科目：微積分　共 1 頁　第 1 頁

本科考試禁用計算器　　＊請在答案卷（卡）內作答

(10%) 1. Please compute the area of the region bounded by the graphs of $f(x) = -x^2 + 6x + 5$ and $g(x) = x^2 + 5$.

(10%) 2. Please compute the volume of the solid of revolution obtained by revolving the region bounded by the curves $f(x) = e^x$ and $g(x) = x^2 + \frac{1}{2}$ from $x = 0$ and $x = 1$ about the x-axis.

(15%) 3. Please show how to use the Newton-Raphson Method to find the roots of the following equation:

$$f(x) = x^4 + x^3 + x^2 + x + 1.$$

Hint: You have to write down the iteration equation.

(15%) 4. Given

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + ... + x^n + ... \quad (-1 < x < 1),$$

please find the Taylor series of $f(x) = \frac{x}{4+9x^2}$ at $x = 0$ and show its convergence interval.

(15%) 5. Please compute

$$\int_0^1 \int_0^{1-x} \sqrt{x+y}(y-2x)^2 dydx.$$

(15%) 6. Please find the minimum and maximum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

(20%) 7. Consider two random variables Y_1 and Y_2 with the joint density function

$$f_{Y_1Y_2}(y_1, y_2) = \begin{cases} 2(1-y_1) & 0 \le y_1 \le 1, \quad 0 \le y_2 \le 1, \\ 0 & \text{elsewhere,} \end{cases}$$

and let $U = Y_1Y_2$. Find the probability density function of U.