國立中央大學104學年度碩士班考試入學試題

所別：通訊工程學系碩士班 不分組(一般生)　科目：工程數學(機率、線性代數)　共 1 頁 第 1 頁
本科考試禁用計算器　*請在答案卷（卡）內作答

參考用

1. (5%) Let $x_1=\begin{bmatrix}1\\2\\3\end{bmatrix}$, $x_2=\begin{bmatrix}3\\1\\1\end{bmatrix}$, $x_3=\begin{bmatrix}-2\\1\\z\end{bmatrix}$. For what value of z will these vectors be linearly independent?

2. (5%) Let $A=\begin{bmatrix}1&0&2\\0&1&0\\2&0&3\end{bmatrix}$ and $B=(I+A)^{-1}(I-A)$. Find the inverse matrix of $(I+B)$.

3. (10%) Let A be an $m\times n$ matrix. Prove that rank(A)=n if and only if A^TA is an invertible $n\times n$ matrix.

4. (10%) Let T: V→ W be a linear transformation. Prove that the $dim(V)=dim(\ Im(T)\ )+dim(\ Ker(T)\ )$ is true, where dim(.), Im(.) and Ker(.) mean the dimension, image and kernel of a subspace (.) respectively.

5. (10%) Let A be an $n\times n$ matrix and λ be an eigenvalue of A.
   (a) (3%) Clearly give the definition of "eigenspace."
   (b) (7%) Prove $A-\lambda I$ is singular.

6. (10%) Use the matrix exponential to solve the initial value problem:

$$Y'=AY,\quad Y(0)=Y_0,\quad \text{where } A=\begin{bmatrix}3&2\\2&3\end{bmatrix} \text{ and } Y_0=\begin{bmatrix}3\\1\end{bmatrix}$$

7.(8%) Box 1 contains 2 black and 8 white balls, and Box 2 contains 2 white and 8 black balls. Two balls are picked from a randomly selected box.
   (a) (4%) Find the probability that both balls are black.
   (b) (4%) If both balls are white, find the probability that they came from Box 2.

8.(10%) The random variable X has C.D.F. $F_X(x)=\begin{cases}0 & x<1,\\ 0.2 & 1\le x<3,\\ 0.6 & 3\le x<4,\\ 0.9 & 4\le x<6,\\ 1 & x\ge 6.\end{cases}$ Define $Y=(X-2)^2$.
   (a) (4%) Find and sketch C.D.F. $F_Y(y)$.
   (b) (6%) Find the expected value and the variance of Y.

9.(10%) Suppose the random variable X has p.d.f. $f_X(x)=kx$, $0<x<2\pi$, and $Y=\sin X$.
   (a) (4%) Find the value of k.
   (b) (6%) Find $f_Y(y)$.

10.(22%) The joint p.d.f. of random variables X and Y is $f_{XY}(x,y)=\begin{cases}\frac{3}{4}y & 0<y<x<2\\ 0 & \text{otherwise}\end{cases}$.
   (a) (6%) Find the p.d.f. of $W=X-Y$.
   (b) (6%) Find the conditional p.d.f. of Y given X.
   (c) (10%) Find the correlation coefficient of X and Y.