1. Solve $y'' + 2y = \begin{cases} 1 & 0 < t < 2 \\ 0 & t > 2 \end{cases}$, $y(0) = y'(0) = 0$ by Laplace transform, and using unit step functions. Please leave the unit step functions in the answer. (10%)

2. If $r(x) = x^2$, $0 < x < 2\pi$, $r(x) = r(x + 2\pi)$

   (a) Find $r(x)$ in the Fourier series (6%)

   (b) Evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = ?$ (3%)

   (c) Evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = ?$ (3%)

   (d) Evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = ?$ (3%)

3. Find the solution for the following ordinary differential equations (ODEs):

   (a) $y' + xy = xy^{-1}$, $y(0) = 3$. (5%)

   (b) $2x \tan y\, dx + \sec^2 y\, dy = 0$ (5%)

   (c) $x^2 y'' - 3xy' + 3y = 3\ln x - 4$, $y(1) = 0$, $y'(0) = 1$ (5%)

4. Referring to the following figure, find the equation that can describes (models) the motion of the mass as a function of time, $y(t)$, and solve the modeling equation if the initial position and velocity of the mass are both zeros, i.e., $y(0) = 0$ and $y'(0) = 0$. (10%)
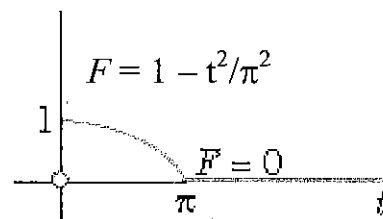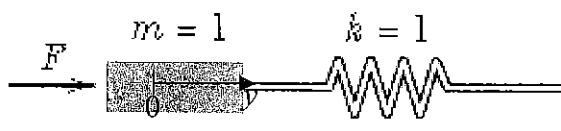
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5. There are nine components of stress states acting on an infinitesimal element as denoted in (a) and can be transformed into the axes of principal stress as (b).

$$(a) \begin{bmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} \quad (b) \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \quad (c) \begin{bmatrix} 70 & 20 & 0 \\ 20 & 35 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (d) \begin{cases} I_1 = 105 \\ I_2 = -2050 \\ I_3 = 170700 \end{cases}$$

The magnitudes of principal stress, σ_p, are the roots of $\sigma_p^3 - I_1\sigma_p^2 - I_2\sigma_p - I_3 = 0$. Using the concept of **_eigenvalues_**, please deduce $I_1 = \sigma_1 + \sigma_2 + \sigma_3, I_2 = -(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)$,

$I_3 = \sigma_1\sigma_2\sigma_3$ (5%). Furthermore, please find the principal stress for the stress state of (c) and (d), respectively (10%)

6. Please find the work required to move a particle by force F($x^2\hat{i} - yz\hat{j} + x\cos(z)\hat{k}$) along the

path C($x = t^2, y = t, z = \pi t; 0 \le t \le 3$) from point Q(9,3,3π) to point P(0,0,0) by calculating the

integral $-\int_C FdR = -\int_0^3 F_x dx + F_y dy + F_z dz$ (10%)

7. The circumference of an ellipse can be given by this formula:

$$Circumference = \pi\sqrt{(a+b)^2},$$

where *a* and *b* are minor and major axes, respectively.
Using this formula, write a **function** that accepts the value of *a* and *b* as parameters from a calling function. The return value of this function should be the circumference of the ellipse. The code is limit to **C**, **C++**, **Visual Basic** or **Fortran** programing language, and please state before your answer. All variables are declared to real numbers. (15%)

8. Fluid flowing through a pipe can flow in a smooth, constant manner, called laminar flow; in a chaotic manner, called turbulent flow; or in an intermediate stage between smooth and turbulent flow, which is called transitional flow. In practice, a value known as the Reynolds number can be used to determine the type of flow. For a Reynolds number below 2000, the flow is laminar and for a Reynolds number above 3000, the flow is turbulent. For a Reynolds number between 2000 and 3000, the flow is transitional.
Using this information, write a program that accepts a Reynolds number as user input and displays a message indicating whether the flow is laminar, turbulent, or transitional. The code is limit to **C**, **C++**, **Visual Basic** or **Fortran** programing language, and please state before your answer. All variables are declared to real numbers. (10%)

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