

*請在答案卷內作答

甲、填充題：共 8 題，每題 8 分，共 64 分。請將答案依題號順序寫在答案卷上。

1. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{2n(x-3)^n}{(n+1)!}$. Answer : _____

2. Evaluate $\int_0^1 \frac{xe^{2x}}{(2x+1)^2} dx$. Answer : _____

3. If $\lim_{x \rightarrow -2} \frac{f(x)}{x^2} = 2$, find the limit: $\lim_{x \rightarrow -2} \left(f(x) + \frac{f(x)}{x} \right)$. Answer : _____

4. Find $\frac{d^{20}}{dx^{20}} \left[\left(\sin \frac{x}{2} \right) \left(\cos \frac{x}{2} \right) \right]$. Answer : _____

5. Find the limit: $\lim_{x \rightarrow \infty} \frac{1}{x \ln x} \int_1^x \ln t dt$. Answer : _____

6. Find an equation of the tangent line to the graph of the function $x^2y^3 - y^2 + xy - 1 = 0$ at $(1, 1)$. Answer : _____

7. What value of a makes $f(x) = x^2 + (a/x)$ have a point of inflection at $x = 1$?
Answer : _____

8. We say that the two commodities are **substitute commodities** if a decrease in the demand for one results in an increase in the demand for the other. Conversely, two commodities are referred to as **complementary commodities** if a decrease in the demand for one results in a decrease in the demand for the other as well. Suppose that the demand functions of two commodities are $x = f(p, q) = \frac{q}{1 + \sqrt{p}}$, and $y = g(p, q) = \frac{3p}{1 + q^4}$. Determine whether these two commodities are substitute, complementary, or neither. Answer : _____

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. Test the series $\sum_{n=0}^{\infty} \frac{e^{2/n}}{n^2}$ for convergence or divergence.

2. Evaluate $\iint_R xe^{y^2} dA$ where R is the plane region bounded by the y -axis, $x = 0$, the horizontal line $y = 4$, and the graph of $y = x^2$.

3. A manufacturer finds that it takes x units of labor and y units of capital to produce $f(x, y) = 100x^{3/4}y^{1/4}$ units of product. If a unit of labor costs \$100, a unit of capital costs \$200, and \$200,000 is budgeted for production, determine how many units should be expended on labor and how many units should be expended on capital in order to maximize production.

參考用