1. Solve the following initial value problems.

   a. $y' + xy = xy^{-1}, \qquad y(0) = 3$

   b. $x^2y'' + 2xy' - 6y = 0, \qquad y(1) = 0.5 \qquad y'(1) = 1.5$

(30%)

2. Find a general solution of the following system. Determine the kind and stability of the critical point.

$$\begin{cases} y_1' = -5y_1 + 5y_2 \\ y_2' = -2y_1 + y_2 \end{cases}$$

(15%)

3. Let $\mathbf{u} = [z \quad x \quad y]$, $\mathbf{v} = [y+z \quad z+x \quad x+y]$, $f = x+y-z$, and $g = xyz$.

   Find the given expressions.

   a. $\nabla(fg)$ b. $\nabla \cdot (\mathbf{u} \times \mathbf{v})$ c. $\nabla \times (g\mathbf{v})$

   d. $\nabla \cdot (\nabla g)$ e. $\nabla \times (g\mathbf{u} + \mathbf{v})$

(15%)

4. Find the eigenvalues and eigenvectors of the following matrix

$$\begin{bmatrix} 6 & 5 & 2 \\ 2 & 0 & -8 \\ 5 & 4 & 0 \end{bmatrix}$$

(15%)

5. Find the Taylor series with center z_0 and its radius of convergence.

   $\cos z, \qquad z_0 = \pi$

(10%)

6. Find the Fourier transform of $f(x)$.

$$f(x) = \begin{cases} 1 & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}, \qquad (0 < x < \pi)$$

(15%)