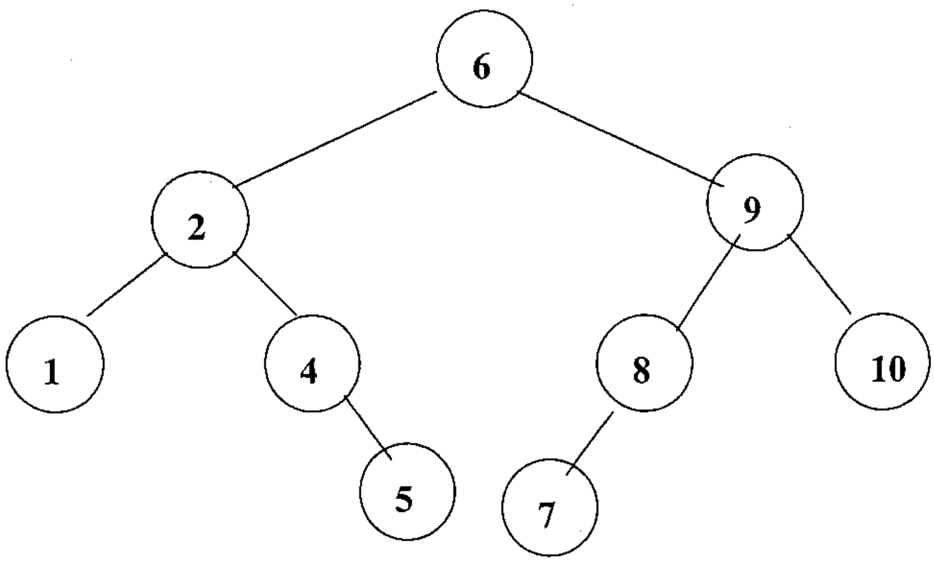
國立中央大學105學年度碩士班考試入學試題

所別： 資工類　　　　共3頁 第1頁

科目： 資料結構與演算法

本科考試禁用計算器　　　　*請在答案卷(卡)內作答

1. Given the following binary search tree.



   A. Give inorder, preorder, and postorder traversal of the above tree. (6 points)
   B. Please draw the resulting tree after inserting node 3 and to the BST above. (3 points)

2. Sequentially insert 1, 2, 3, 4, and 5 into an empty AVL tree. Draw the resulting AVL tree after each insertion. Notice some insertions may trigger AVL tree rotations. (10 points)
   PS: You are supposed to draw 5 AVL trees here.

3. Given the sequence of numbers: 5, 4, 3, 2, 1, we intend to sort them into ascending order.
   A. How many swaps does it take to sort this sequence via Insertion sort? (4 points)
   B. How many recursive calls does it take by the Merge sort to get the job done? (4 points)
   C. How many times the partition procedure will be called by the Quicksort to do the same job? (4 points)

4. Given the sequence of numbers: 15, 24, 33, 42, 51, we intend to sort them into ascending order.
   A. What is the required storage space for the counting sort? (4 points)
   B. What is the result of the first pass if the radix sort is used? (4 points)
   C. Why does the radix sort need to use a stable sort inside the loop? (4 points)

注意：背面有試題

5. Given the sequence of numbers: 5, 4, 3, 2, 1.
   A. Insert them into an empty B-tree of degree 3. Draw the resulting B-tree. (4 points)
   B. In which kind of applications is B-tree frequently used? (3 points)

6. Assume it is a known fact that problem X is NP-hard.
   A. How can we prove that problem Y is also NP-hard by taking advantage of polynomial-time reduction and the fact? (6 points)
   B. How can we further prove that Y is NP-complete? (6 points)

7. Below is the description of the Minimum Edit Cost (MEC) problem, described as follows. Assuming string $A=a_1a_2...a_m$ and string $B=b_1b_2...b_n$, the MEC problem is to find a sequence of edit operations such that the total cost of the operations in the sequence is minimized. There are three types of operations of costs 1, 2 and 3, as shown below.

   Operation 1: Deletion of a character from A **(cost 1)**

   Operation 2: Insertion of a character into A **(cost 2)**

   Operation 3: Substitution of a character in A with another character **(cost 3)**

   The MEC problem can be solved by a dynamic programming algorithm, which stores in the array element $c[i,j]$ the minimum cost of operations to transform substring $a_1a_2...a_i$ of A to substring $b_1b_2...b_j$ of B, where $0 \le i \le m$ and $0 \le j \le n$. The recursive relations and the boundary conditions of $c[i, j]$ are the core of the dynamic programming algorithm. Write down:
   A. The recursive relation of $c[i,j]$ (8 points)
   B. The boundary conditions of $c[i,j]$ (6 points)

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8. Below is the Bellman-Ford Shortest Path Algorithm (BFSPA). The input of the BFSPA is a weighted directed graph $G=(V, E)$ and a specific source node s, where

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V is the node set and E is the edge set and every edge in E has a negative or positive weight. For every node x in V-{s}, the BFSPA can derive the shortest path from s to x and output the sum of weights of all edges on the path. Modify the algorithm to determine if the given graph has a negative-weighted cycle. The output of your modified algorithm is "YES" if the graph has a negative-weighted cycle; "NO", otherwise. You should write down the whole modified algorithm including the input, the output and all steps. Note that the BFSPA uses indentation to represent the block structure. So please use the indentation properly when writing the modified BFSPA algorithm. (12 points)

Algorithm BFSPA

Input: A weighted digraph G=(V, E), a source node s and a two dimension array w, where V is the node set, E is the edge set, and $w[x][y]$ is the weight of the edge (x, y)

Output: An array d, where $d[x]$ represents the sum of weights of all edges on the shortest path from s to x, a node in V-{s}

```
d[s]←0;
for every node x in V-{s}
   d[x]←∞
for i←1 to |V|-1 do
   for every edge (x, y) in E do
      If  d[y] > d[x] + w[x][y]  then
          d[y]← d[x] + w[x][y]
return d
```

9. Given a knapsack with capacity C, and n objects $o_1,\ldots,o_n$ with weights $w_1,\ldots,w_n$ and profits $p_1,\ldots,p_n$, the fractional knapsack problem is to determine x_i ($0 \le x_i \le 1$, $1 \le i \le n$) such that $\sum_{1\le i\le n} x_i w_i \le C$ and $P=\sum_{1\le i\le n} x_i p_i$ is maximized. Write a greedy algorithm to solve the fractional knapsack problem to output $x_1,\ldots,x_n$, and P. You should write down the complete algorithm including the input, the output and all steps. (12 points)