國立中央大學105學年度碩士班考試入學試題

所別： 電機工程學系 碩士班 電波組(一般生)　　共1頁 第1頁

科目： 工程數學(不含複變)

本科考試禁用計算器　　＊請在答案卷(卡)內作答

一、There is a transformation $T(x_1, x_2)=(3x_1+x_2, 2x_1-5x_2)$.

(1) **(10%)** Find the matrix of T relative to the bases $B=\{(1,2),(-1,-2)\}$ and

$B'=\{(2,-1),(-1,2)\}$.

(2) **(10%)** Find $[T(v)]_{B'}$, where $v=\begin{bmatrix}6\\3\end{bmatrix}_B$. Note: $v=\begin{bmatrix}6\\3\end{bmatrix}_B$ means the vector v is on the basis B.

二、**(15%)** Please prove that S can be a basis of a matrix $M_{2\times2}$, where $S=\left\{\begin{bmatrix}2&0\\0&3\end{bmatrix},\begin{bmatrix}1&4\\0&1\end{bmatrix},\begin{bmatrix}0&1\\3&2\end{bmatrix},\begin{bmatrix}0&1\\2&0\end{bmatrix}\right\}$.

三、(1) **(10%)** Find the least squares solution of the following system $Ax=b$, where

$$A=\begin{bmatrix}1&1\\1&2\\1&3\end{bmatrix},\ b=\begin{bmatrix}0\\1\\3\end{bmatrix}.$$

(2) **(10%)** From (1), find the orthogonal projection of b on the column space of A.

四、**(15 %)** Solve the following differential equation (Show the details of your work)

$$3y^2y'+3x^2y^3=e^{-x^3}\cosh x.$$

五、**(15%)** Find the Laplace transform of the following function (Show the details of your work):

$f(t)=\frac{k}{p}t$ if $0<t<p$, $f(t+p)=f(t)$ and k is a constant.

六、(1) **(5%)** Find the continuous-time non-periodic signal $x(t)$ with its Fourier transform

$$X(j\omega)=\frac{1}{j\omega+500}.$$

(2) **(10%)** Find the continuous-time non-periodic signal $x(t)$ with its Fourier transform

$$X(j\omega)=\frac{5(j\omega)-100}{(j\omega)^2+100(j\omega)-120000}.$$