

國立中央大學 105 學年度碩士班考試入學試題

所別： 環境工程研究所碩士班 甲組(一般生)

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環境工程研究所碩士班 乙組(一般生)

科目： 工程數學

本科考試禁用計算器

*請在答案卷(卡)內作答

1. (20%) Please show that

$$\mathcal{L}(e^{at}f(t)) = F(s - a),$$

$$\mathcal{L}(t^n f(t)) = (-1)^n \frac{d^n F(s)}{ds^n}.$$

Solve the initial value problem $y'' + 2y' + y = t \cdot e^{-t}$, $y(0) = 1$, $y'(0) = 2$ using Laplace transform.

2. (25%) Describe in words the similarities and differences between scalars and vectors.

Consider the following velocity vectors:

(i). $\mathbf{q} = 2x^2\hat{i} - 2yz\hat{j} - (y^2 + 3)\hat{k}$

(ii). $\mathbf{u} = \hat{i} - 4\hat{j}$

(iii). $\mathbf{v} = 2y\hat{i} + 2x^2y^2\hat{j} - 2x^2yz\hat{k}$

(iv). $\mathbf{w} = z^3\hat{k}$

Determine:

(a) $\mathbf{q} \cdot \mathbf{u}$

(b) $\mathbf{v} \times \mathbf{w}$

(c) $|\mathbf{q}|$

(d) show the gradient, divergence, and curl of \mathbf{q} and \mathbf{v} . In each case, state whether your answer is a scalar or vector. What is your justification?

3. (30%) Consider the following 2-step reaction system:



Please obtain the expressions of concentrations as a function of time

for A, B and C. (Hint: $\frac{d[A]}{dt} = -k_A[A]$).

注意：背面有試題

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4. (25%) An industrial process operates on a batch basis and produces a liquid waste at the rate of 100 gallon/day. It is desired to design a stirred batch reactor to treat the waste by chemical oxidation with KMnO_4 . Preliminary bench studies indicate a first-order decay of the waste so that $r(C) = -kC$ where $k = 3.5/\text{day}$. It takes 4 hours to empty the treated waste products from the tank at the end of a reaction period and refill the tank with a new batch of waste. Surplus 50 gallons tanks are available as treatment vessels. Please determine the number of 50 gallons vessels required to treat 100 gallon/day if it is desired that the waste concentration be reduced to 10% of its original value in each treatment. (Hint: $\ln(10) \cong 2.3$)

注意：背面有試題