

*請在答案卷內作答

1. Find a general solution of the following equation.

$$y' = e^{2x-1}y^2 \quad (15\%)$$

2. Solve the following initial value problems.

$$y'' - 4y' + 3y = 10 \cos x, \quad y(0) = 1, \quad y'(0) = -1 \quad (15\%)$$

3. Find eigenvalues and eigenfunctions of the following problem.

$$y'' + \lambda y = 0, \quad y(0) = y'(L) = 0 \quad (15\%)$$

4. Let $f = zy + yx$, $\mathbf{v} = [y \ z \ 4z - x]$, $\mathbf{w} = [y^2 \ z^2 \ x^2]$. Find

- a. ∇f b. $\nabla \cdot \mathbf{v}$
 c. $\nabla \times \mathbf{w}$ d. $\nabla^2 f^2$
 e. $\nabla \cdot (\mathbf{v} \times \mathbf{w})$

(15%)

5. Find the Laplace transforms of the following functions

$$\cos^2 t \quad (10\%)$$

6. Solve the following problem by Gauss elimination.

$$\begin{cases} 4y + 3z = 8 \\ 2x - z = 2 \\ 3x + 2y = 5 \end{cases} \quad (15\%)$$

7. Find solutions of the following equations by separation of variables.

$$xu_{xy} + 2yu = 0 \quad (15\%)$$

