科目 微積分 類組別 A2 951 共 1 頁 第 1 頁

＊請在答案卷內作答

甲、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. Sketch the region of integration, reverse the order of integration, and evaluate the integral. $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\,dydx$.

2. Determine whether the series is convergent or divergent. Give your reason.
   (a) $\sum_{n=1}^{\infty} \frac{n}{3n^2+5}$ (b) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$.

3. Find the points on the sphere $x^2+y^2+z^2=4$ that are colsest to and farthest from the point $(3, 1-1)$.

乙、填充題：共 8 題，每題 8 分，共 64 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Suppose that the first derivative of $y=f(x)$ is $y'=6x(x+1)(x-2)$. At what points does the graph of f have a point of inflection? Answer : _____

2. Evaluate the integral. $\int_2^6 x\sqrt{2x-3}\,dx$. Answer : _____

3. Find the slope of the tangent line to the graph of $2xy+e^{x+y}-2=0$ at the point $P(0, \ln 2)$. Answer : _____

4. Find the limit: $\lim_{h\to 0} \frac{1}{h}\int_x^{x+h} \sqrt{1+t^2}\,dt$. Answer : _____

5. Find the limit: $\lim_{n\to\infty} \sum_{j=1}^{n} \frac{\sqrt{n^2-j^2}}{n^2}$. Answer : _____

6. Find the area under the curve $y=\frac{\ln x}{x^2}$ from $x=1$ to $x=\infty$. Answer : _____

7. A new drug is introduced through an advertising campaign to a population of 1 million potential customers? The rate at which the population hears about the drug is assumed to be proportional to the number of people who are not yet aware of the drug? By the end of 1 year) half of the population has heard of the drug? How many will have heard of it by the end of 2 years? Answer : _____

8. Find the absolute maximum value of the function $f(x,y)=x^2-2xy+2y$ on the rectangle $D=\{(x,y)|\,0\le x\le 3,\ 0\le y\le 2\}$. Answer : _____