國立中央大學106學年度碩士班考試入學試題

所別： 機械工程學系 碩士班 製造與材料組(一般生)
機械工程學系光機電工程 碩士班 光機組(一般生)
能源工程研究所 碩士班 不分組(一般生)　　　　共1頁　第1頁

科目： 工程數學

本科考試可使用計算器，廠牌、功能不拘　須有計算過程　　*請在答案卷　內作答

參考用

1. Solutions for ordinary differential equations (ODEs) (25%)

(a) Find the solution for the ODE $e^{3\theta}(dr+3rd\theta)=0$ **(5%)**

(b) Find the solution for the ODE $y''+16y=4\sin t, y(0)=0, y'(0)=1$ **(10%)**

(c) Find a basis of solutions by the Frobenius method of the following ODE: $x^2y''+x(2x-1)y'+(x+1)y=0$. **(10%)**

2. Series solution and Linear algebra (25%)

(a) For a general **Legendre** equation $(1-x^2)y''-2xy'+n(n+1)y=0$ Please derive the recursion relation. **(7%)** and use power series method to solve it as n=1. **(8%)**

(b) Please use **Cramer's rule** to evaluate A_n and B_n of the following equations **(10%)**

$$\begin{cases}(25-n^2)A_n+0.05nB_n=\dfrac{4}{n^2\pi}\\ -0.05nA_n+(25-n^2)B_n=0\end{cases}$$

3. Laplace transform / Fourier analysis (25%)

(1) Fourier series expansion of a periodic function (or signal) $f(t)$ can be represented as

$$f(t)=\tfrac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nw_0t+\sum_{n=1}^{\infty}b_n\sin nw_0t,$$

where $a_0=\frac{2}{T}\int_T f(t)dt$, $a_n=\frac{2}{T}\int_T f(t)\cos n\omega_0tdt$, and $b_n=\frac{2}{T}\int_T f(t)\sin n\omega_0tdt$.

(i) **(6%)** Give the physical meaning of $a_0/2$ and ω_0.

(ii) **(8%)** If now $g_1(t)$ is an **even** function, please address and rewrite its Fourier series expansion; likewise, if $g_2(t)$ is an **odd** function, what does the form of its Fourier series expansion become?

(2) If a measured temperature can be characterized as a square wave of amplitude *20℃* and period *T = 6 sec*,

(i) **(3%)** please first **sketch the time waveform of measured temperature** stated as the above in the form of an **even function** ***or*** an **odd function** (Time in second: -∞ ↔ ∞);

(ii) **(8%)** then, derive its Fourier series expansion (till **the first six terms**).

4. Partial differential equations (PDEs) (25%)

(a) **(10%)**

$y_{tt}=a^2y_{xx}$

B.C.: y(0,t) =0, y(2,t)=0

I.C.: y(x,0)=f(x), $y_t(x,0)=0$

Solve y(x,t)

(b) **(15%)**

$y_{tt}=a^2y_{xx}$+g(x)

B.C.: y(0,t) =u1, y(L,t)=u2

I.C.: y(x,0)=φ (x), $y_t(x,0)=\psi(x)$

(Hint: you can set y(x,t) = V(x,t)+u(x), or use other methods)

Solve y(x,t)