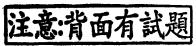1. (10 points) Let $\{X_t: t = 0, 1, 2, \dots\}$ be a stochastic process. Suppose that $X_{t+1} = 2X_t$ with probability $1 - p$ and $X_{t+1} = 0.5X_t$ with probability p.

   (a) Compute $E(X_{t+9}|X_t)$

   (b) $\{X_t\}$ is said to be a martingale if $E(X_{t+1}|X_0, \dots, X_t) = X_t$. Determine the value of p such that $\{X_t: t = 0, 1, 2, \dots\}$ is a martingale.

2. (26 points) Suppose that we would like to test if a given coin is fair or not. We then toss the coin 10 times. The rejection rule is proposed as follows: reject the hypothesis that it is a fair coin if 8 heads or 8 tails are observed out of 10 tosses; do not reject the fair coin hypothesis, otherwise. Given the proposed rejection rule:

   (a) Compute the type I error of the test.

   (b) Now suppose it is known that the coin is biased, with the probability of landing head up being 0.9, compute the power of this test.

3. (14 points) If $Y_1, Y_2, \dots, Y_n, \dots$ are i.i.d. normal distribution $N(0, \sigma^2)$. Show *asymptotic* distributions of $\left(\dfrac{\overline{Y}}{\frac{s}{\sqrt{n}}}\right)$ and $\left(\dfrac{\sum_{i=1}^{n} Y_i^2}{s^2\sqrt{n}}\right) - \sqrt{n}$, where $\overline{Y} = \dfrac{\sum_{i=1}^{n} Y_i}{n}$ and $s^2 = \dfrac{\sum_{i=1}^{n}(Y_i^2 - \overline{Y})^2}{(n-1)}$.

4. (50 points) The following ordinary least square regression analysis was conducted:

$$Earnings_i = \beta_0 + \beta_1 age_i + \beta_2 age_i^2 + \beta_3 PartTime_i + \beta_4(age_i \times PartTime_i) + \varepsilon_i,$$

where *Earnings*= worker i's earnings, *age* = worker i's age, $age^2 = age \times age$, *PartTime* =1 if i works part time; =0, otherwise, and $\varepsilon_i \sim N(0, \sigma^2)$. The computer printout for the regression is shown below:

Predictor	Coefficient	95% confidence interval	
Constant	3.22	2.38	4.06
age	0.08	0.05	0.12
age^2	-.001	-.0015	-.0006
PartTime	-.41	-1.11	.279
age × *PartTime*	-.006	-.021	.008

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Analysis of Variance

Source	df	SS
Model	-	606.2
Residual	3698	-
Total	-	24515.4

Based on the regression results, answer the following questions.

(a) (10 points) Residual variation of *Earnings* = ? $\overline{R}^2$ = ?

(b) (10 points) Conduct the F test for the goodness of fit for the model. State the corresponding null and alternative hypotheses and make the conclusion. (α =0.05) (Note: $F_{0.025}(4,3698) \cong 2.80$, $F_{0.05}(4,3698) \cong 2.38$, $F_{0.025}(5,3698) \cong 2.58$, $F_{0.05}(5,3698) \cong 2.22$.)

(c) (10 points) What is the marginal effect of the variable "*age*" for a part time worker? At what age will a part-timer obtain his (or her) highest earnings? How about that for a full-time worker? Explain your findings.

(d) (10 points) Is the following statement true, false or uncertain? Explain. (Please provide detail explanations.)
"*The variables "PartTime" and "age × PartTime" should be deleted from the model, because their coefficient estimates are statistically nonsignificant.*"

(e) (10 points) Now suppose σ^2 increases with worker's age. How would this relationship influence the OLS estimators? (Please provide detail explanations.)

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