所別：　大氣科學學系大氣物理 碩士班 不分組(一般生)
　　　　大氣科學學系大氣物理 碩士班 不分組(在職生)

科目：　應用數學　　問答題及計算題，計算題需計算過程，無計算過程者不予計分

本科考試禁用計算器

1. (a) Find a second-order homogeneous linear ODE for which the given functions are solutions. (b) Show linear independence by the Wronskian. (c) Solve the initial value problem.

$$x^2, x^2 \ln x, \qquad y(1) = 4, \ y'(1) = 6$$

(15%)

2. Solve the following initial value problems.

$$y_1' = y_1 + 3y_2$$
$$y_2' = \tfrac{1}{3}y_1 + y_2$$

$$y_1(0) = 12, \qquad y_2(0) = 2$$

(15%)

3. Solve the following integral equation

$$y(t) - \int_0^t y(\tau)(t - \tau)d\tau = 2 - \tfrac{1}{2}t^2$$

(15%)

4. Find an eigenbasis and diagonalize the following matrix.

$$\begin{bmatrix} -1 & 2 & -2 \\ 2 & 4 & 1 \\ 2 & 1 & 4 \end{bmatrix}, \qquad \lambda_1 = 5$$

(15%)

5. Let $\mathbf{u} = \begin{bmatrix} z & x & y \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} y+z & z+x & x+y \end{bmatrix}$, $f = x + y - z$, and $g = xyz$.

Find (a) $\nabla \cdot (\nabla(fg))$, (b) $\nabla \times (g\mathbf{v})$, and (c) $\nabla \cdot (\mathbf{u} \times \mathbf{v})$

(15%)

6. Find its Fourier series of $f(x)$ as given over one period.

$$f(x) = \begin{cases} 0 & \text{if } -1 < x < 0 \\ x & \text{if } \ \ 0 < x < 1 \end{cases}$$

(15%)

7. Show that for the completely insulated bar, $u_x(0,t) = 0$, $u_x(L,t) = 0$, $u(x,0) = f(x)$ and separation of variables gives the following solution,.

$$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\frac{n\pi x}{L} e^{-(cn\pi/L)^2 t}.$$

Find A_0 and A_n.

(10%)