1. Reduce to first order and solve(計算題，應詳列計算過程，無計算過程者不予計分)。

$$xy'' + 2y' + xy = 0, \qquad y_1 = \cos x/x$$

(15%)

2. Solve the following initial value problem(計算題，應詳列計算過程，無計算過程者不予計分)。

$$y'' + 4y = 8x^2, \qquad y(0) = -3, \qquad y'(0) = 0$$

(15%)

3. Solve the following boundary value problem(計算題，請詳列計算過程，無計算過程者不予計分)。

$$y'' - 2y' + (\lambda + 1)y = 0, \quad y(0) = 0, \qquad y(1) = 0$$

(15%)

4. Solve the following problem by Laplace transform(計算題，請詳列計算過程，無計算過程者不予計分)。

$$x\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t} = xt,$$

$$w(x,0) = 0 \ \text{ if } x \geq 0, \qquad w(0,t) = 0 \ \text{ if } t \geq 0$$

(20%)

5. Find the corresponding Fourier series of the following periodic function(計算題，請詳列計算過程，無計算過程者不予計分)。

$$f(x) = \cos(\pi x), \quad -\tfrac{1}{2} < x < \tfrac{1}{2}, \quad f(x+1) = f(x)$$

(20%)

6. Find the eigenvalues and eigenvectors of the following matrix(計算題，請詳列計算過程，無計算過程者不予計分)。

$$\begin{bmatrix} 0 & -4 & -8 \\ 4 & 0 & -8 \\ 8 & 8 & 0 \end{bmatrix}$$

(15%)