科目 微積分 類組別 A3 A4 A7 共 2 頁 第 1 頁

甲、填充題：共 8 題，每題 8 分，共 64 分。請在答案卷上列出題號依序作答。

請注意：本（甲、）部分，共 8 題，命題型態為填充題，僅將答案依題號順序依序寫在答案卷第一頁即可，不需列出計算過程。計算過程與其結論將視為試算部分，不予計分。

1. Evaluate $\lim_{x\to 0^+}(\sin x)^x$. Answer :_____

2. Let $f(x)=\int_1^x \sqrt{1+t^2}\,dt$. Find $(f^{-1})'(0)$. Answer :_____

3. Find the average value of the function $f(x,y)=e^{-x^2}$ over the plane region R where R is the triangle with vertices $(0,0),(1,0)$, and $(1,1)$. Answer :_____

4. Evaluate the definite integral $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$. Answer :_____

5. Find the direction in which the function $f(x,y)=x^2y+e^{xy}\sin y$ increases most rapidly at the point $(1,0)$. Answer :_____

6. Find the maximum value of $f(x,y)=x^2+2y^2-2x+3$ subject to the constraint $x^2+y^2=10$. Answer :_____

7. Find the work done by the force $\mathbf{F}=\frac{x\mathbf{i}+y\mathbf{j}}{(x^2+y^2)^{3/2}}$ over the plane curve $\mathbf{r}(t)=(e^t\cos t)\mathbf{i}+(e^t\sin t)\mathbf{j}$ from the point $(1,0)$ to the point $(e^{2\pi},0)$. Answer :_____

8. Convert the integral $\int_0^{2\pi}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} 3\,dz\,r\,dr\,d\theta$, $r\geq 0$ to an equivalent integral in spherical coordinates. Answer :_____

注意:背面有試題

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. Determine if the series converges or diverges.

   a. (6 分) $\sum_{n=0}^{\infty} e^{-n^2}$ b. (6 分) $\sum_{n=1}^{\infty} \sin\frac{1}{n}$

2. Use the limit definition to show that $g'(0)$ exists but $g'(0) \neq \lim_{x\to 0} g'(x)$, where

$$g(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

3. Find the area of the surface cut from the paraboloid $x^2 + y + z^2 = 2$ by the plane $y = 0$.

注意:背面有試題