國立中央大學 109 學年度碩士班考試入學試題

所別： 光電類　　共 2 頁　第 1 頁

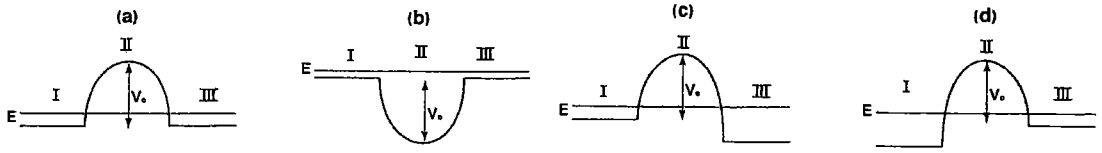
科目： 近代物理

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷(卡)內作答

Boltzmann constant k=8.617×10^{-5} eV/K
Electron mass m_e=0.511 MeV/c^2
Planck's constant h=4.136×10^{-15} eV · s

＊計算題需計算過程，無計算過程者不予計分

1. (10 pts) An electron is injected into a potential energy bump from the left to right. The energy of the electron is E and E < |V_0| where V_0 is the barrier height of the potential energy bump. If the electron is to emerge in ***Region III*** with a shorter wavelength than that in ***Region I***, the appropriate potential step is given by which of the following?



2. (10 pts) A pendulum, consisting of a frictionless pivot, a massless rod, and a massive bod, is a harmonic oscillator. Assume a pendulum completes one round trip in 1 sec. According to quantum physics, calculate its minimum energy in Joules. [Please assume all physical parameters that you need, for example, the mass of the bob, length of the pendulum, amplitude and so on.]

3. (10 pts) An electron is trapped in an infinite potential well of length L and ground state energy E_1. At t = 0, the wavefunction is: $\psi(x,0) = \frac{1}{\sqrt{5L}}(\phi_1(x) + 2\phi_2(x))$, where $\phi_1(x)$ and $\phi_2(x)$ are normalized wavefunctions in the ground and first excited states, respectively. Sketch the wavefunction, $\psi(x,t)$, at $t = \frac{h}{2E_1}$.

4. (10 pts) A photon of wavelength λ = 440 nm imparts all of its energy to a free electron in a metal. Suppose the separation potential between the electron and the metal's surface is 1 volt. What will be the wavelength of the electron upon emergence from the metal?

注意：背面有試題

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5. (10 pts) According the hydrogen atom wave function, we know that the radial probability density for $n = 1$, $l = 0$ has its maximum at $r = a_0$, where a_0 is the Bohr radius. Prove that the maximum probability for $n = 2$, $l = 1$ state is at $4a_0$. The wave functions are listed below:

For $(n, l, m_l) = (1, 0, 0)$: $\Theta(\theta) = \frac{1}{\sqrt{2}}, \Phi(\phi) = \frac{1}{\sqrt{2\pi}}, \mathrm{R(r)} = \frac{2}{{a_0}^{3/2}} e^{-r/a_0}$

For $(n, l, m_l) = (2, 1, 0)$: $\Theta(\theta) = \sqrt{\frac{3}{2}}\cos\theta, \Phi(\phi) = \frac{1}{\sqrt{2\pi}}, \mathrm{R(r)} = \frac{1}{\sqrt{3}(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$

where $\mathbf{\Phi}(\phi)$, $\mathbf{\Theta}(\theta)$ and $\mathbf{R}(r)$ is the azimuthal function, polar function and radial function, respectively.

6. (10 pts) Consider classical oscillators (total number N) in the cavity at temperature T, calculate the average energy of the oscillators. Hint: the energies of oscillators are distributed according to Maxwell-Boltzmann distribution, proportional to $e^{-E/kT}$.

7. (10 pts) A photon of energy E interacts with an electron at rest and undergoes pair production as follows,

$$\text{photon} + e^- \rightarrow e^+ + e^- + e^-$$

If the two electrons and positron move off with identical momenta in the direction of the initial photon, calculate the energy of photon and kinetic energy of three final particles.

8. (10 pts) (a) Derive the relation between group velocity and phase velocity as a function of wavelength. (b) When white light travels through glass, is $V_{group} > V_{phase}$, or $V_{phase} > V_{group}$? Why?

9. (10 pts) Consider an oxygen atom, the electronic configuration is $1s^2 2s^2 2p^4$ in the ground state. What is the largest possible value of the total m_s of 8 electrons? If adding the z components of the intrinsic spins of the 4 electrons in the $2p$ subshell, what is the maximum total z component of the intrinsic spin?

10. (10 pts) A certain metal has a Fermi energy of 5.00 eV. Find the number of electrons per unit volume with energy between 5.00 eV and 5.10 eV for T=300 K.

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