國立中央大學110學年度碩士班考試入學試題

所別： 統計研究所 碩士班 不分組(一般生) 共2頁 第1頁
統計研究所 碩士班 不分組(在職生)

科目： 基礎數學

本科考試可使用計算器，廠牌、功能不拘 *請在答案卷(卡)內作答

※計算題需計算過程，無計算過程者不予計分

1. (20%) Given $\int_0^\infty f(x)dx = 1$, $\int_0^\infty g(y)dy = 1$, and $\int_0^\infty \int_0^\infty h(x,y)dxdy = 1$ where f, g, and h are nonnegative functions, verify the following inequalities

   (a) (10%) (Jensen's inequality) Verify that if ψ is a convex function
   $$\psi\left(\int_0^\infty xf(x)dx\right) \le \int_0^\infty \psi(x)f(x)dx.$$

   (b) (10%) Verify
   $$\int_0^\infty \int_0^\infty \log\left(\frac{h(x,y)}{f(x)g(y)}\right) h(x,y)dxdy \ge 0.$$
   (Hint: Use Jensen's inequality)

2. (20%)

   (a) (10%) Compute
   $$\int_{-\infty}^0 \int_{-y}^\infty \frac{1}{2\pi} e^{-\frac{x^2+y^2}{2}} dxdy$$

   (b) (10%) Show that
   $$\int_1^\infty \int_y^\infty \frac{1}{2\pi} e^{-\frac{x^2+y^2}{2}} dxdy \le \frac{1}{2\sqrt{\pi}}.$$
   Hint: Show $\int_y^\infty e^{-\frac{x^2}{2}}dx \le \frac{1}{y}e^{-\frac{y^2}{2}}$ for all $y \ge 0$.

3. (20%) Give a full rank matrix
   $$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{pmatrix},$$
   and $H = A(A^TA)^{-1}A^T$ where A^T is the transpose of A and A^{-1} is the inverse of A.

   (a) (10%) Find the eigenvalue for H^2. In addition, verify H^2 is positive semi-definite and
   $$trace[H^2] = rank[H^2]$$

   (b) (10%) Find the eigenvalue for $I - H$. In addition, verify $I - H$ is positive semi-definite and
   $$trace[I - H] = rank[I - H]$$
   where I is the identity matrix.

注意:背面有試題

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4. (20%) A matrix of the form is $H = I - \theta \frac{VV^T}{V^TV}$ where I is identity and V is a non-zero column vector. V^T is the transpose of v. θ is a constant.
   (a) (10%) Evaluate H^2 and obtain possible θ to achieve $H^2 = I$.
   (b) (10%) Evaluate HV and obtain possible θ to achieve $HV = -V$.
5. (10%) Find the minimum distance from a point on the surfaces $x + y + z = c$ with some constant c to the origin.
6. (10%) Given the two by two matrix

$$G = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Evaluate $e^G = \sum_{n=0}^{\infty} \frac{1}{n!} G^n$ using the eigenvalue decomposition $G = B\Lambda B^{-1}$. Note that $G^0 = I$ where I is the identity matrix.

注意:背面有試題