國立中央大學110學年度碩士班考試入學試題

所別： 統計研究所 碩士班 不分組(一般生)　　共2頁 第1頁

統計研究所 碩士班 不分組(在職生)

科目： 數理統計

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷(卡)內作答

※計算題需計算過程，無計算過程者不予計分

1. Let the joint probability density function of (X,Y) be

$$f(x,y)=1,\quad 0<x<1,\ x<y<x+1.$$

(a) Find the marginal probability density functions of X and Y. (8%)

(b) Find the covariance of X and Y. (8%)

2. Let $X_1<X_2<\cdots<X_n$ be the order statistics of n independent observations from a $U(0,1)$ distribution and let Y_n be $Y_n=X_n-X_1$. Please calculate the expectation of Y_n as $n\to\infty$. (15%)

3. One observation, X, is taken from the normal distribution $N(0,\sigma^2)$

(a) Find an unbiased estimator of σ^2. (5%)

(b) Find the maximum likelihood estimator of σ. (9%)

4. Let X be a single observation from the probability density function

$$f(x)=\theta x^{\theta-1};\quad x\in(0,1)\text{ and }\theta>0.$$

Let $Y=-(\log X)^{-1}$.

(a) Find the probability density function of Y. (7%)

(b) If the set $\left[\frac{y}{2},\ y\right]$ is the confidence interval of θ, please calculate its confidence coefficient. (8%)

注意:背面有試題

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5. Let $X_1, X_2, ..., X_n$ be a random sample from the normal distribution $N(\theta, 1)$.

   (a) Show that the uniformly minimum variance unbiased estimator (UMVUE) of θ^2 is $\bar{X}^2 - \frac{1}{n}$. (15%)

   (b) If $\mathrm{Var}\left(\bar{X}^2 - \frac{1}{n}\right) = \frac{2}{n^2} + \frac{4\theta^2}{n}$. Does the variance of $\bar{X}^2 - \frac{1}{n}$ attain the Cramér-Rao lower bound for estimating θ^2? Give your reason. (10%)

6. Let p equal the proportion of Americans who have brothers or sisters. Suppose a random sample of $n = 1,600$ Americans yielded $y = 1,280$ who had brothers or sisters.

   (a) Give an estimate for the proportion p. (5%)

   (b) Please construct an approximate 95% confidence interval for the proportion p based on the collected data. (10%) Hint: $z_{0.025} = 1.96$ and $z_{0.05} = 1.645$

注意:背面有試題